Factoring Polynomials of One Variable

Procedure for Factoring Binomials

I. Factor out the Greatest Common Monomial Factor. Choose the sign of the GCMF so that the remaining binomial has a positive leading coefficient.

II. If the remaining binomial is second degree or higher, try to factor it using one of the two methods below. Attempt the methods in the order given.

1. If the remaining binomial is a difference of two squares, then factor it into the product of two binomials using the special case formula

   \[ a^2 - b^2 = (a - b)(a + b) \]

2. If the remaining binomial is a sum or difference of two cubes, then factor it into the product of a binomial and a trinomial using one of the two special case formulas

   \[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]
   \[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]

III. If step II produced any binomials of second degree or higher, then apply step II again to see if they can be factored further. Otherwise you are finished factoring. If the binomial is still in its original form then it is prime.

Procedure for Factoring Trinomials

I. Factor out the Greatest Common Monomial Factor. Choose the sign of the GCMF so that the remaining trinomial has a positive leading coefficient.

II. Try to factor the remaining trinomial using one of the three methods below. Attempt the methods in the order given.

1. If the remaining trinomial is a perfect square, then factor it into a binomial squared using one of the two special case formulas

   \[ a^2 + 2ab + b^2 = (a + b)^2 \]
   \[ a^2 - 2ab + b^2 = (a - b)^2 \]

2. If the remaining trinomial has the form

   \[ x^2 + Bx + C \]

   then list all integral factor pairs of \( C \) such that the factor with the largest absolute value in each pair has the sign of \( B \), and the other factor has the sign of \( BC \).

   If any pair of factors, \( F_1 \) and \( F_2 \), has a sum equal to \( B \), then write the trinomial as the product of the two binomials

   \[ (x + F_1)(x + F_2) \]

   If no pair of factors has a sum equal to \( B \), then the remaining trinomial is prime.

3. If the remaining trinomial has the form

   \[ Ax^2 + Bx + C \]

   then list all integral factor pairs of \( AC \) such that the factor with the largest absolute value in each pair has the sign of \( B \), and the other factor has the sign of \( BC \).

   If any pair of factors, \( F_1 \) and \( F_2 \), has a sum equal to \( B \), then write the trinomial as the quadrinomial

   \[ Ax^2 + F_1x + F_2x + C \]

   and use the Procedure for Factoring Quadrinomials to factor it into the product of two binomials.

   If no pair of factors has a sum equal to \( B \), then the remaining trinomial is prime.

III. If step II produced any binomials of second degree or higher, then use the Procedure for Factoring Binomials to see if they can be factored further.

   Otherwise you are finished factoring. If the trinomial is still in its original form then it is prime.

Procedure for Factoring Quadrinomials

I. Factor out the Greatest Common Monomial Factor. Choose the sign of the GCMF so that the remaining quadrinomial has a positive leading coefficient.

II. Try using the grouping method to factor the remaining quadrinomial into the product of two binomials.

1. Group the four terms of the remaining quadrinomial into two pairs.

2. Factor out the GCMF from each pair of terms, creating two terms that are each the product of a monomial and a binomial. Choose the signs of the GCMFs so that the remaining binomials have positive leading coefficients.

3. If the binomial factors in each remaining term are identical then factor them out of both terms.

If the quadrinomial doesn’t factor with the first grouping, try a second time with a different grouping.

III. If step II produced any binomials of second degree or higher, then use the Procedure for Factoring Binomials to see if they can be factored further.

   Otherwise you are finished factoring. If the quadrinomial is still in its original form then it is prime.