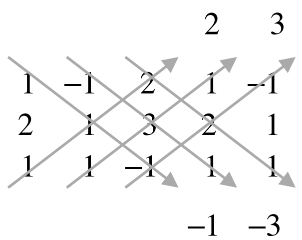
Disclaimer

You should use this practice exam to assess your speed and to improve your ability to correctly identify different problem types. The questions on this practice exam are taken from exams given in previous semesters, but they may not be representative of the questions that will appear on this semester's exam. You should also invest time re-reading the relevant parts of your textbook, reviewing your notes, and practicing homework problems.

Math 8: Chapter 8 Practice Exam **Answers**

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. Evaluate the determinant $\begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & 1 & -1 \end{vmatrix}$. (10 points)



$$\Rightarrow \left[-1+(-3)+4\right]-\left[2+3+2\right]=0-7=-7$$

2. Use Cramer's rule to solve the system $\begin{cases} 3x-7y=2 \\ 2x-3y=-1 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 3 & -7 & 2 \\ 2 & -3 & -1 \end{array}\right]$. (10 points)

$$D=\begin{vmatrix} 3 & -7 \\ 2 & -3 \end{vmatrix}=(3)(-3)-(2)(-7)=-9-(-14)=5$$

$$D_x=\begin{vmatrix} 2 & -7 \\ -1 & -3 \end{vmatrix}=(2)(-3)-(-1)(-7)=-6-7=-13$$

$$D_y=\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix}=(3)(-1)-(2)(2)=-3-4=-7$$

$$x=\frac{D_x}{D}=-\frac{13}{5} \qquad y=\frac{D_y}{D}=-\frac{7}{5}$$

So the solution is the point $\left(-\frac{13}{5},-\frac{7}{5}\right)$.

3. Write the system of equations as an augmented matrix and solve the system using matrices (row operations). (12 points)

$$\begin{cases} x - y = 1 \\ -x - 2y + 3z = 2 \\ 4y - 3z = 3 \end{cases}$$

$$\begin{cases} x - y = 1 \\ -x - 2y + 3z = 2 \\ 4y - 3z = 3 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ -1 & -2 & 3 & 2 \\ 0 & 4 & -3 & 3 \end{array}\right] \quad R_2 = r_1 + r_2$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & -3 & 3 & 3 \\ 0 & 4 & -3 & 3 \end{array}\right] \quad R_2 = (-1/3)r$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 4 & -3 & 3 \end{array}\right] \quad \begin{array}{l} R_1 = r_1 + r_2 \\ \\ R_3 = r_3 - 4r_2 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 7 \end{array}\right] \quad \begin{array}{l} R_1 = r_1 + r_3 \\ R_2 = r_2 + r_3 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 7 \end{array}\right]$$

So the solution is the point (7, 6, 7).

4. Let $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 0 \\ 7 \end{bmatrix}$. (13 points)

a. Calculate $\mathbf{A}(\mathbf{B}+\mathbf{C})$.

$$\begin{aligned}
\mathbf{A}(\mathbf{B}+\mathbf{C}) &= \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}\left(\begin{bmatrix} 3 \\ -2 \end{bmatrix}+\begin{bmatrix} 0 \\ 7 \end{bmatrix}\right) \\
&= \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} \\
&= \begin{bmatrix} (1)(3)+(2)(5) \\ (-1)(3)+(0)(5) \end{bmatrix} \\
&= \begin{bmatrix} 13 \\ -3 \end{bmatrix}
\end{aligned}$$

b. Calculate $\mathbf{AB}+\mathbf{AC}$.

$$\begin{aligned}
\mathbf{AB}+\mathbf{AC} &= \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix}+\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 7 \end{bmatrix} \\
&= \begin{bmatrix} (1)(3)+(2)(-2) \\ (-1)(3)+(0)(-2) \end{bmatrix}+\begin{bmatrix} (1)(0)+(2)(7) \\ (-1)(0)+(0)(7) \end{bmatrix} \\
&= \begin{bmatrix} -1 \\ -3 \end{bmatrix}+\begin{bmatrix} 14 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} 13 \\ -3 \end{bmatrix}
\end{aligned}$$

c. Does the distributive property, $\mathbf{A}(\mathbf{B}+\mathbf{C}) = \mathbf{AB}+\mathbf{AC}$, hold true for these matrices?

Yes. Both sides produce the same matrix, $\begin{bmatrix} 13 \\ -3 \end{bmatrix}$.

5. Use the elimination method to solve the system $\begin{cases} x+\frac{5}{4}y=1 \\ \frac{8}{3}x-5y=1 \end{cases} \Rightarrow \begin{cases} 4x+5y=4 \\ 8x-15y=3 \end{cases}$. (10 points)

$$\begin{cases} 4x+5y=4 \\ 8x-15y=3 \end{cases} \Rightarrow \begin{array}{r} 12x+15y=12 \\ 8x-15y=3 \\ \hline 20x=15 \\ x=\frac{15}{20} \\ x=\frac{3}{4} \end{array}$$

$$\begin{cases} 4x+5y=4 \\ 8x-15y=3 \end{cases} \Rightarrow \begin{array}{r} -8x-10y=-8 \\ 8x-15y=3 \\ \hline -25y=-5 \\ y=\frac{-5}{-25} \\ y=\frac{1}{5} \end{array}$$

So the solution is the point $\left(\frac{3}{4},\frac{1}{5}\right)$.

6. Solve the system of equations by substitution: $\begin{cases} 3x+2y=2 \\ 2x+2y=3 \end{cases}$ (10 points)

$$\begin{aligned} 2x+2y&=3 \\ 2x&=3-2y \\ x&=\frac{3}{2}-y \end{aligned}$$

$$\begin{aligned} 3x+2y&=2 \\ 3\left(\frac{3}{2}-y\right)+2y&=2 \\ \frac{9}{2}-3y+2y&=2 \\ \frac{9}{2}-y&=2 \\ -y&=-\frac{5}{2} \\ y&=\frac{5}{2} \end{aligned}$$

$$\begin{aligned} x&=\frac{3}{2}-\left(\frac{5}{2}\right) \\ x&=-1 \end{aligned}$$

So the solution is the point $\left(-1,\frac{5}{2}\right)$.

7. Let $\mathbf{M} = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$. (10 points)

a. Find $\mathbf{M}^{-1}$.

If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\mathbf{A}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\mathbf{M}^{-1} = \frac{1}{(2)(-2) - (1)(-6)}\begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix}$$
$$= \frac{1}{-4+6}\begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

b. Use $\mathbf{M}^{-1}$ to solve the system $\begin{cases} 2x - 6y = -18 \\ x - 2y = -7 \end{cases} \Rightarrow \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -18 \\ -7 \end{bmatrix}$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{M}^{-1}\begin{bmatrix} -18 \\ -7 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -18 \\ -7 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} (-2)(-18) + (6)(-7) \\ (-1)(-18) + (2)(-7) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -6 \\ 4 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$