

**Math 8: Exam 5**

**You will not receive full credit if you do not clearly show how you are obtaining your answers. Show all work on this exam; do not attach other work. Circle your answers.**

1. Write the equation  $\log_6 \frac{1}{36} = -2$  in its equivalent exponential form. (3 points)

2. Find the inverse of the one-to-one function  $f(x) = -\frac{1}{2}x + 1$ . (6 points)

3. An initial deposit of \$5,000 grows at an annual rate of 8.5% for 5 years. (10 points)

a. Find the final balance resulting from annual compounding.  $A = P(1 + r)^t$

b. Find the final balance resulting from continuous compounding.  $A = Pe^{rt}$

4. Write the equation  $\sqrt[3]{8} = 2$  in its equivalent logarithmic form. (3 points)

5. An artifact originally had 40 grams of carbon-14 present. The decay model  $A = 40e^{-0.000121t}$  describes the amount of carbon-14 present,  $A$ , in grams after  $t$  years. Use this model to solve the problems below. (12 points)

a. How many grams of carbon-14 will be present after 19,030 years?

b. How old will the artifact be when only 20 grams of carbon-14 remain?

6. Use properties of logarithms to expand each logarithmic expression as much as possible. (6 points)

a.  $\log_3 \sqrt{\frac{x}{yz^2}}$

b.  $\ln\left(\frac{x^3 \sqrt{y}}{z}\right)$

8.  $f(x) = 3x + 2$  and  $g(x) = 2x - 3$

(8 points)

a. Find  $(f \circ g)(x)$

b. Find  $(g \circ f)(x)$

- c. Are these functions inverses of each other?

8. If  $f(x) = 2^x + 1$  then its inverse is  $f^{-1}(x) = \log_2(x - 1)$ . (20 points)

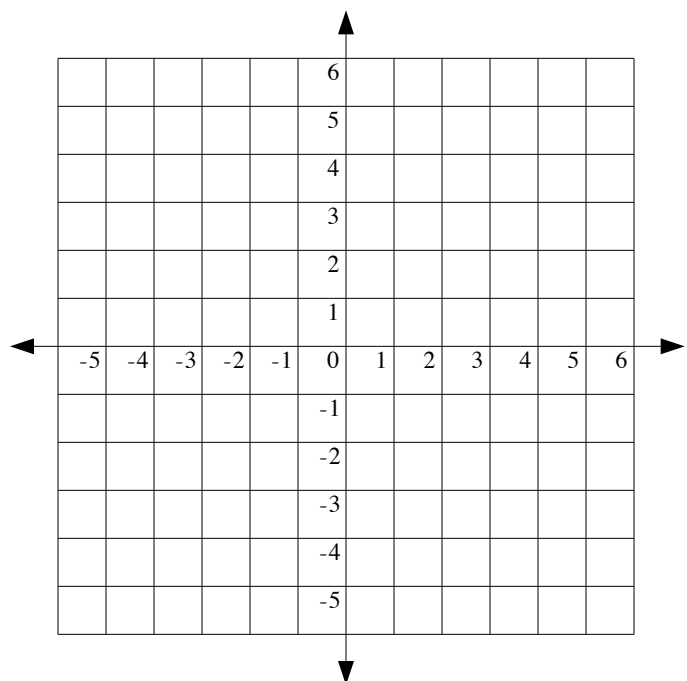
a. Find the coordinates of at least four points for each function. Plot  $f(x)$  and  $f^{-1}(x)$  on the coordinate plane provided. Be sure to graph the asymptote for each function.

$x$	$f(x)$

$x$	$f^{-1}(x)$

b. Write the domain and range of  $f(x)$  in interval notation.

c. Write the domain and range of  $f^{-1}(x)$  in interval notation.



9. Solve each logarithmic equation.

(6 points)

a.  $x = \log_6 \sqrt{6}$

b.  $\log_7 x = -2$

10. Solve the exponential equation  $2^x = 3^{x-1}$ . Round the solution to two decimal places.

(10 points)

- 11.** Use properties of logarithms to condense each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. (6 points)

a.  $\log x + \frac{1}{2}\log y - 4\log z$

b.  $2\ln x - (3\ln y + \ln z)$

- 12.** Solve the logarithmic equation  $\log_4 x + \log_4(3x + 8) = 2$ . Be sure to reject any value of  $x$  that produces the logarithm of a negative number or the logarithm of 0. (10 points)