Math 8: Exam 1 **Solutions**

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. Add, subtract, or multiply as directed. (16 points)
Express your answer as a single polynomial in standard form.

a. $(3x-4)^2$

$$(3x-4)^2 = (3x)^2 - 2(3x)(4) + (4)^2$$
$$= 9x^2 - 24x + 16$$

b. $(7t-3)(4t+8)$

$$(7t-3)(4t+8) = (7t)(4t) + (7t)(8) + (-3)(4t) + (-3)(8)$$
$$= 28t^2 + 56t - 12t - 24$$
$$= 28t^2 + 44t - 24$$

c. $(5x+2y)(5x-2y)$

$$(5x+2y)(5x-2y) = (5x)^2 - (2y)^2$$
$$= 25x^2 - 4y^2$$

d. $7(2x^3 - 5x^2 - 3) - 4(4x^3 + 9x - 8)$

$$7(2x^3 - 5x^2 - 3) - 4(4x^3 + 9x - 8) = 14x^3 - 35x^2 - 21 - 16x^3 - 36x + 32$$
$$= -2x^3 - 35x^2 - 36x + 11$$

2. Use synthetic division to find the quotient and remainder when $2x^4 - 3x^2 + 2$ is divided by $x-2$. (5 points)

2	2	0	–3	0	2
		4	8	10	20
	2	4	5	10	22

So the quotient is $2x^3 + 4x^2 + 5x + 10$ and the remainder is 22.

3. Simplify each expression. Assume that all variables are positive when they appear. (15 points)

a. $\sqrt[3]{16x^4} - x \cdot \sqrt[3]{2x}$

$$\begin{aligned}\sqrt[3]{16x^4} - x \cdot \sqrt[3]{2x} &= \sqrt[3]{8x^3 \cdot 2x} - x \cdot \sqrt[3]{2x} \\ &= \sqrt[3]{8x^3} \cdot \sqrt[3]{2x} - x \cdot \sqrt[3]{2x} \\ &= 2x \cdot \sqrt[3]{2x} - x \cdot \sqrt[3]{2x} \\ &= x \cdot \sqrt[3]{2x}\end{aligned}$$

b. $\dfrac{(xy)^{1/4}(x^2y^2)^{1/2}}{(x^2y)^{3/4}}$

$$\begin{aligned}\frac{(xy)^{1/4}(x^2y^2)^{1/2}}{(x^2y)^{3/4}} &= \frac{x^{1/4}y^{1/4} \cdot x^{2(1/2)}y^{2(1/2)}}{x^{2(3/4)}y^{3/4}} \\ &= \frac{x^{1/4}y^{1/4} \cdot xy}{x^{3/2}y^{3/4}} \\ &= \frac{x^{5/4}y^{5/4}}{x^{3/2}y^{3/4}} \\ &= \frac{y^{1/2}}{x^{1/4}}\end{aligned}$$

c. $\dfrac{2-\sqrt{5}}{2+3\sqrt{5}}$

$$\begin{aligned}\frac{2-\sqrt{5}}{2+3\sqrt{5}} &= \frac{(2-\sqrt{5})}{(2+3\sqrt{5})} \cdot \frac{(2-3\sqrt{5})}{(2-3\sqrt{5})} \\ &= \frac{(2)^2-(2)(3\sqrt{5})-(2)(\sqrt{5})+(\sqrt{5})(3\sqrt{5})}{(2)^2-(3\sqrt{5})^2} \\ &= \frac{4-6\sqrt{5}-2\sqrt{5}+3 \cdot 5}{4-9 \cdot 5} \\ &= \frac{4-8\sqrt{5}+15}{4-45} \\ &= \frac{19-8\sqrt{3}}{-41} \\ &= \frac{8\sqrt{3}-19}{41}\end{aligned}$$

4. Factor completely each polynomial. If the polynomial cannot be factored, say it is *prime*. (20 points)

a. $3 - 27x^2$

$$3 - 27x^2 = 3(1 - 9x^2)$$
$$= 3(1 - 3x)(1 + 3x)$$

b. $9x^2 - 12x + 4$

$$9x^2 - 12x + 4 = (3x)^2 - 2(3x)(2) + (2)^2$$
$$= (3x - 2)^2$$

c. $10x^2 - 7x - 6$

$$10x^2 - 7x - 6 = 10x^2 - 12x + 5x - 6$$
$$= 2x(5x - 6) + 1(5x - 6)$$
$$= (2x + 1)(5x - 6)$$

d. $x^3 - 3x^2 - x + 3$

$$x^3 - 3x^2 - x + 3 = x^2(x - 3) - 1(x - 3)$$
$$= (x^2 - 1)(x - 3)$$
$$= (x - 1)(x + 1)(x - 3)$$

e. $64 - 27x^3$

$$64 - 27x^3 = (4)^3 - (3x)^3$$
$$= (4 - 3x)[(4)^2 + (4)(3x) + (3x)^2]$$
$$= (4 - 3x)(16 + 12x + 9x^2)$$

5. Use synthetic division to determine whether $x + 4$ is a factor of $x^6 - 16x^4 + x^2 - 16$. (5 points)

−4	1	0	−16	0	1	0	−16
		−4	16	0	0	−4	16
	1	−4	0	0	1	−4	0

Since the remainder is zero, $x + 4$ is a factor of $x^6 - 16x^4 + x^2 - 16$.

6. Perform the indicated operation and simplify the result. (20 points)
Leave your answer in factored form.

a. $\dfrac{12}{x^2+x}\cdot\dfrac{x^3+1}{4x^2-4x+4}$

$$\frac{12}{x^2+x}\cdot\frac{x^3+1}{4x^2-4x+4}=\frac{12}{x(x+1)}\cdot\frac{(x+1)(x^2-x+1)}{4(x^2-x+1)}$$
$$=\frac{3\cdot 4(x+1)(x^2-x+1)}{x\cdot 4(x+1)(x^2-x+1)}$$
$$=\frac{3}{x}$$

b. $\dfrac{1-\dfrac{x}{x+1}}{2-\dfrac{x-1}{x}}$

$$\frac{1-\dfrac{x}{x+1}}{2-\dfrac{x-1}{x}}=\frac{\left(1-\dfrac{x}{x+1}\right)}{\left(2-\dfrac{x-1}{x}\right)}\cdot\frac{x(x+1)}{x(x+1)}$$
$$=\frac{x(x+1)-x^2}{2x(x+1)-(x-1)(x+1)}$$
$$=\frac{x^2+x-x^2}{(x+1)(2x-(x-1))}$$
$$=\frac{x}{(x+1)(2x-x+1)}$$
$$=\frac{x}{(x+1)(x+1)}$$
$$=\frac{x}{(x+1)^2}$$

6. *Continued from the previous page.*

c. $\dfrac{2x-3}{x^2+8x+7}-\dfrac{x-2}{(x+1)^2}$

$$\begin{aligned}\frac{2x-3}{x^2+8x+7}-\frac{x-2}{(x+1)^2} &= \frac{2x-3}{(x+7)(x+1)}-\frac{x-2}{(x+1)(x+1)} \\ &= \frac{(2x-3)}{(x+7)(x+1)}\cdot\frac{(x+1)}{(x+1)}-\frac{(x-2)}{(x+1)(x+1)}\cdot\frac{(x+7)}{(x+7)} \\ &= \frac{2x^2-x-3}{(x+7)(x+1)(x+1)}-\frac{x^2+5x-14}{(x+7)(x+1)(x+1)} \\ &= \frac{2x^2-x-3-x^2-5x+14}{(x+7)(x+1)(x+1)} \\ &= \frac{x^2-6x+11}{(x+7)(x+1)(x+1)}\end{aligned}$$

7. Find the quotient and remainder when $2x^4-3x^3+x+1$ is divided by $2x^2+x+1$. (5 points)

$$\begin{array}{r} x^2-2x+\frac{1}{2} \\ 2x^2+x+1\overline{\big)\,2x^4-3x^3+0x^2+x+1} \\ \underline{2x^4+x^3+x^2\quad\quad\quad} \\ -4x^3-x^2+x\quad \\ \underline{-4x^3-2x^2-2x\quad} \\ x^2+3x+1 \\ \underline{x^2+\frac{1}{2}x+\frac{1}{2}} \\ \frac{5}{2}x+\frac{1}{2} \end{array}$$

So the quotient is $x^2-2x+\frac{1}{2}$ and the remainder is $\frac{5}{2}x+\frac{1}{2}$.

8. Simplify each expression. (10 points)

a. $\left(\dfrac{3x^{-2}}{4y^{-2}}\right)^{-3}$

$$\left(\frac{3x^{-2}}{4y^{-2}}\right)^{-3} = \frac{3^{-3}x^{6}}{4^{-3}y^{6}}$$
$$= \frac{4^{3}x^{6}}{3^{3}y^{6}}$$
$$= \frac{64x^{6}}{27y^{6}}$$

b. $\dfrac{9x^{-2}(yz)^{-1}}{3^{3}x^{4}y}$

$$\frac{9x^{-2}(yz)^{-1}}{3^{3}x^{4}y} = \frac{9x^{-2}y^{-1}z^{-1}}{3^{3}x^{4}y}$$
$$= \frac{9}{27x^{4}y\cdot x^{2}y^{1}z^{1}}$$
$$= \frac{1}{3x^{6}y^{2}z}$$

9. Find the value of $\left|\,|4x|-|5y|\,\right|$ when $x = 3$ and $y = -2$. (4 points)

$$\left|\,|4x|-|5y|\,\right| = \left|\,|4(3)|-|5(-2)|\,\right|$$
$$= \left|\,|12|-|-10|\,\right|$$
$$= |12-10|$$
$$= |2|$$
$$= 2$$