Math 8: Exam 2 **Solutions**

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. **a**. Solve the inequality $1 < 1 - \frac{1}{2}x < 4$. (10 points)

$$1 < 1 - \frac{1}{2}x < 4$$
$$0 < -\frac{1}{2}x < 3$$
$$(-2)(0) > (-2)\left(-\frac{1}{2}x\right) > (-2)(3)$$
$$0 > x > -6$$
$$-6 < x < 0$$

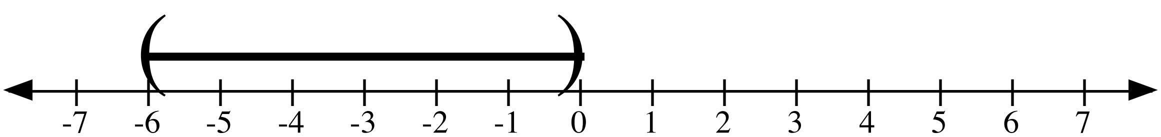
b. Express your answer using set notation.

$\{x \mid -6 < x < 0\}$

c. Express your answer using interval notation.

$(-6, 0)$

d. Graph the solution set.



2. Write the expression, $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2$ in the standard form $a + bi$. (6 points)

$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 = \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right) + \left(\frac{\sqrt{3}}{2}i\right)^2 = \frac{1}{4} + \frac{\sqrt{3}}{2}i + \frac{3}{4}i^2 = \frac{1}{4} + \frac{\sqrt{3}}{2}i - \frac{3}{4} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

3. Find the real solutions of the equation, $x^6 + 7x^3 - 8 = 0$. (10 points)

$$x^6 + 7x^3 - 8 = 0 \qquad \text{Let } u = x^3$$
$$u^2 + 7u - 8 = 0$$
$$(u-1)(u+8) = 0$$

$u - 1 = 0$ or $u + 8 = 0$

$u = 1$ $\quad$ $u = -8$

$x^3 = 1$ $\quad$ $x^3 = -8$

$\sqrt[3]{x^3} = \sqrt[3]{1}$ $\quad$ $\sqrt[3]{x^3} = \sqrt[3]{-8}$

$x = 1$ $\quad$ $x = -2$

4. Solve the equation, $3x^2 + x - \frac{1}{2} = 0$, by completing the square. (12 points)

$$3x^2 + x = \frac{1}{2}$$
$$x^2 + \frac{1}{3}x = \frac{1}{6}$$
$$x^2 + \frac{1}{3}x + \left(\frac{1}{6}\right)^2 = \frac{1}{6} + \left(\frac{1}{6}\right)^2$$
$$\left(x + \frac{1}{6}\right)^2 = \frac{6}{36} + \frac{1}{36}$$
$$\left(x + \frac{1}{6}\right)^2 = \frac{7}{36}$$
$$\sqrt{\left(x + \frac{1}{6}\right)^2} = \pm\sqrt{\frac{7}{36}}$$
$$x + \frac{1}{6} = \pm\frac{\sqrt{7}}{6}$$
$$x = -\frac{1}{6} \pm \frac{\sqrt{7}}{6}$$
$$x = \frac{-1 \pm \sqrt{7}}{6}$$

5. A bathroom tub will fill in 15 minutes with both faucets open and the stopper in place. (12 points)
With both faucets closed and the stopper removed, the tub will empty in 20 minutes.
How long will it take for the tub to fill if both faucets are open and the stopper is removed?

Let x = the time needed to fill the tub with both faucets running and the drain unstoppered.

	Time	Rate	Work
faucets open	15	1/15	1 tub
drain open	20	-1/20	-1 tub
together	x	$1/x$	1 tub

The rates combine so...

$$\frac{1}{15} - \frac{1}{20} = \frac{1}{x}$$
$$60x\left(\frac{1}{15} - \frac{1}{20}\right) = 60x\left(\frac{1}{x}\right)$$
$$4x - 3x = 60$$
$$x = 60 \text{ minutes}$$

It will take 60 minutes to fill the tub with both faucets running and the drain unstoppered.

6. Write the expression, $\frac{10}{3-4i}$ in the standard form $a + bi$. (6 points)

$$\frac{10}{3-4i} = \frac{10}{(3-4i)} \cdot \frac{(3+4i)}{(3+4i)} = \frac{10(3+4i)}{(3)^2+(4)^2} = \frac{10(3+4i)}{9+16} = \frac{10(3+4i)}{25} = \frac{2(3+4i)}{5} = \frac{6+8i}{5} = \frac{6}{5} + \frac{8}{5}i$$

7. **a**. Solve the inequality, $|2x-3| \geq 2$. (10 points)

$$|2x-3| \geq 2$$

$$2x-3 \geq 2 \qquad \text{or} \qquad 2x-3 \leq -2$$

$$2x \geq 5 \qquad \qquad 2x \leq 1$$

$$x \geq \frac{5}{2} \qquad \qquad x \leq \frac{1}{2}$$

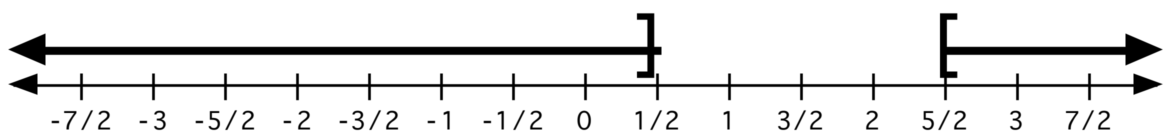
b. Express your answer using set notation.

$$\left\{x \,\middle|\, x \leq \frac{1}{2} \text{ or } x \geq \frac{5}{2}\right\}$$

c. Express your answer using interval notation.

$$\left(-\infty, \frac{1}{2}\right] \cup \left[\frac{5}{2}, \infty\right)$$

d. Graph the solution set.



8. Solve the equation, $\frac{2}{y+3} + \frac{3}{y-4} = \frac{5}{y+6}$. (10 points)

$$(y+3)(y-4)(y+6)\left(\frac{2}{y+3} + \frac{3}{y-4}\right) = (y+3)(y-4)(y+6)\left(\frac{5}{y+6}\right)$$

$$2(y-4)(y+6) + 3(y+3)(y+6) = 5(y+3)(y-4)$$

$$2(y^2+2y-24) + 3(y^2+9y+18) = 5(y^2-y-12)$$

$$2y^2 + 4y - 48 + 3y^2 + 27y + 54 = 5y^2 - 5y - 60$$

$$5y^2 + 31y + 6 = 5y^2 - 5y - 60$$

$$31y + 6 = -5y - 60$$

$$36y = -66$$

$$y = -\frac{66}{36}$$

$$y = -\frac{11}{6}$$

This solution checks out.

9. Find the real solutions of the equation, $\sqrt{2x+3}-\sqrt{x+1}=1$. (12 points)

$$\sqrt{2x+3}-\sqrt{x+1}=1$$
$$\sqrt{2x+3}=1+\sqrt{x+1}$$
$$\left(\sqrt{2x+3}\right)^2=\left(1+\sqrt{x+1}\right)^2$$
$$2x+3=1+2\sqrt{x+1}+(x+1)$$
$$2x+3=x+2+2\sqrt{x+1}$$
$$x+1=2\sqrt{x+1}$$
$$(x+1)^2=\left(2\sqrt{x+1}\right)^2$$
$$x^2+2x+1=4(x+1)$$
$$x^2+2x+1=4x+4$$
$$x^2-2x-3=0$$
$$(x-3)(x+1)=0$$

$x-3=0$ or $x+1=0$

$x=3$ $\qquad$ $x=-1$

Checking shows that both 3 and -1 are solutions..

10. Find the real solutions, if any, of the equation, $\frac{3}{5}x^2-x=\frac{1}{5}$. Use the quadratic formula. (12 points)

$$\frac{3}{5}x^2-x=\frac{1}{5}$$
$$(5)\left(\frac{3}{5}x^2-x\right)=\left(\frac{1}{5}\right)(5)$$
$$3x^2-5x=1$$
$$3x^2-5x-1=0$$

$a=3$

$b=-5$

$c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-1)}}{2(3)}$$
$$x=\frac{5\pm\sqrt{25+12}}{6}$$
$$x=\frac{5\pm\sqrt{37}}{6}$$