Math 8: Exam 3 **Solutions**

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. Let $f(x) = \begin{cases} 3x+6 & \text{if } x < -2 \\ \sqrt{x+2} & \text{if } x \geq -2 \end{cases}$ (16 points)

a. Evaluate $f(2)$

$$f(2) = \sqrt{(2)+2} = \sqrt{4} = 2$$

b. Evaluate $f(-2)$

$$f(-2) = \sqrt{(-2)+2} = \sqrt{0} = 0$$

c. Evaluate $f(-4)$

$$f(-4) = 3(-4)+6 = -12+6 = -6$$

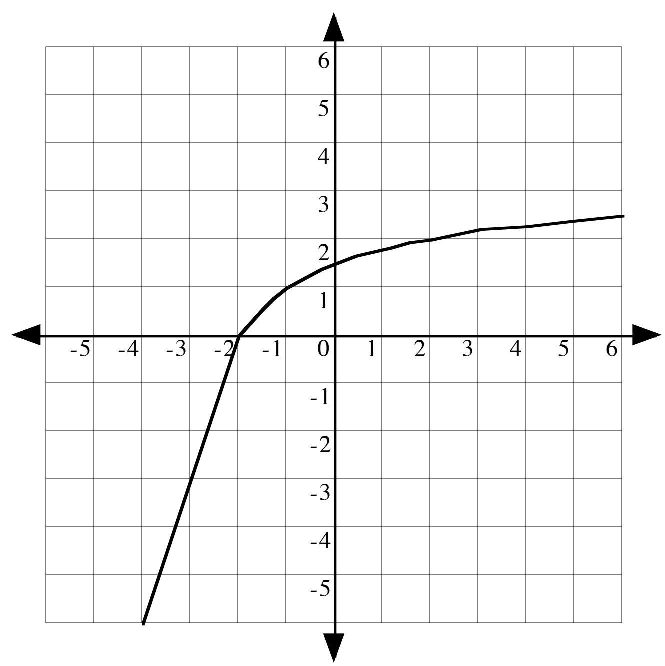
d. Evaluate $f(-1)$

$$f(-1) = \sqrt{(-1)+2} = \sqrt{1} = 1$$

e. Evaluate $f(-3)$

$$f(-3) = 3(-3)+6 = -9+6 = -3$$

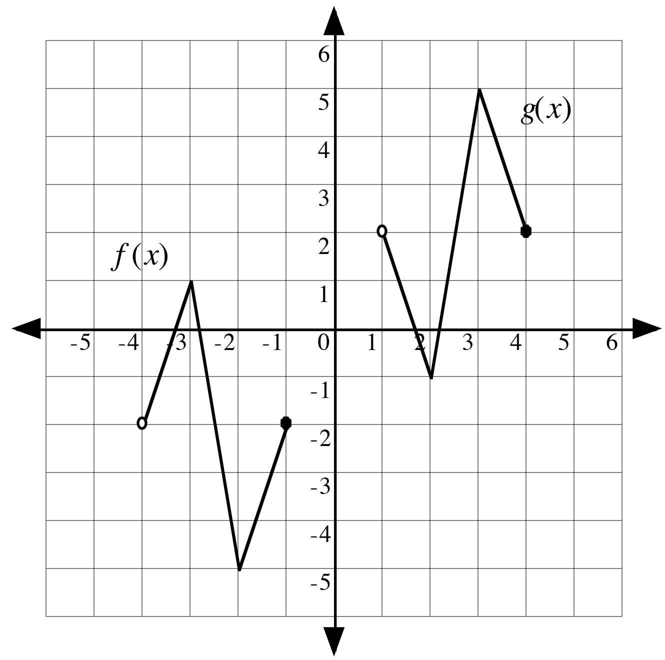
g. Graph $f(x)$ on the coordinate plane provided.



2. Given the graph of $f(x)$ shown below, graph $g(x) = -f(x-5)$. (12 points)
Explain in words how the graph of $g(x)$ is obtained from the graph of $f(x)$.

Subtracting 5 from the independent variable $f(x)$ translates the graph five units to the right.

Multiplying the dependent variable $f(x)$ by -1 reflects the graph about the x-axis.



3. Let $h(x) = x^2 - 2x$. (12 points)

a. Find the average rate of change from 2 to 4.

$$\frac{h(4)-h(2)}{4-2} = \frac{[(4)^2 - 2(4)] - [(2)^2 - 2(2)]}{2} = \frac{8-0}{2} = \frac{8}{2} = 4$$

b. Find the equation of the secant line containing $(2, h(2))$ and $(4, h(4))$.

$m = 4$

$(x_1, y_1) = (2, h(2)) = (2,0)$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - h(2) &= m(x-2) \\ y - 0 &= 4(x-2) \\ y &= 4x - 8 \end{aligned}$$

c. Determine whether $h(x)$ is *even*, *odd*, or *neither*.

$h(-x) = (-x)^2 - 2(-x) = x^2 + 2x$

So $h(x)$ is *neither* odd nor even.

4. Let $y = \dfrac{x^2 - 4}{2x}$. (12 points)

a. List the x-intercepts of the relation. If there are no x-intercepts then write *no x-intercepts*.

Set $y = 0$.

$$
\begin{aligned}
0 &= \frac{x^2 - 4}{2x} \\
0 &= x^2 - 4 \\
4 &= x^2 \\
\pm 2 &= x
\end{aligned}
$$

So the x-intercepts are $(2,0),(-2,0)$.

b. List the y-intercepts of the relation. If there are no y-intercepts then write *no y-intercepts*.

Set $x = 0$ $\quad y = \dfrac{(0)^2 - 4}{2(0)} = \dfrac{-4}{0}$ which is undefined

So there are *no y-intercepts*.

c. Test the relation for symmetry about the x-axis, y-axis and origin.

x-axis symmetry (change y into -y)

$$
\begin{aligned}
-y &= \frac{x^2 - 4}{2x} \\
y &= \frac{4 - x^2}{2x}
\end{aligned}
$$

y-axis symmetry (change x into -x)

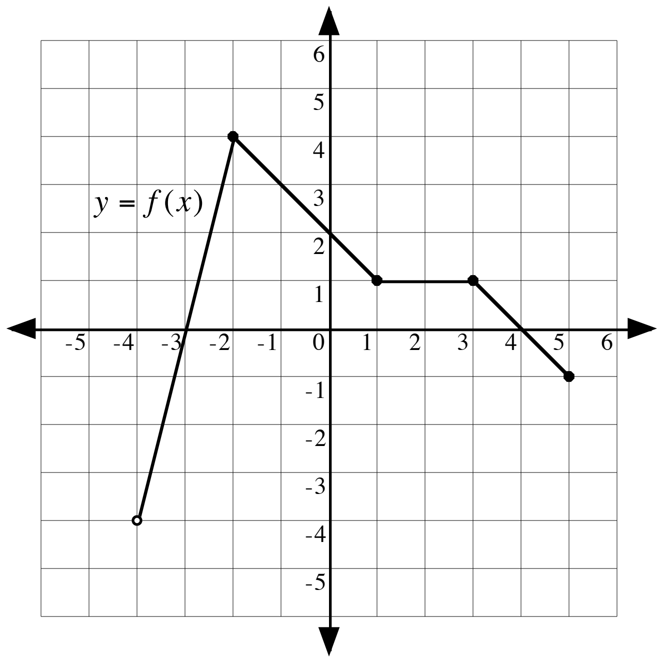
$$
y = \frac{(-x)^2 - 4}{2(-x)} = \frac{x^2 - 4}{-2x} = -\frac{x^2 - 4}{2x}
$$

origin symmetry (change x into -x and y into -y)

$$
\begin{aligned}
-y &= \frac{(-x)^2 - 4}{2(-x)} \\
-y &= -\frac{x^2 - 4}{2x} \\
y &= \frac{x^2 - 4}{2x}
\end{aligned}
$$

So the relation is symmetric about the origin.

5. Use the graph of $f(x)$ shown below to determine the indicated characteristics. (18 points)



a. Determine the function's domain.

$(-4, 5]$

b. Determine the function's range.

$(-4, 4]$

c. Determine the function's *x*-intercepts, if any.

$(-3, 0), (4, 0)$

d. Determine the function's *y*-intercept, if any.

$(0, 2)$

e. Determine the intervals on which the function is increasing, if any.

$(-4, -2)$

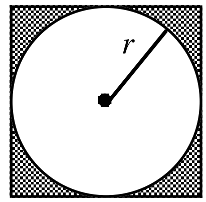
f. Determine the intervals on which the function is decreasing, if any.

$(-2, 1) \cup (3, 5)$

g. Determine the intervals on which the function is constant, if any.

$(1, 3)$

6. A circle of radius r is inscribed in a square. See the figure. (8 points)



a. Express the area A of the square as a function of the radius r of the circle.

$$A(r) = l^2$$
$$A(r) = (2r)^2$$
$$A(r) = 4r^2$$

b. Express the perimeter p of the square as a function of r.

$$p(r) = 4l$$
$$p(r) = 4(2r)$$
$$p(r) = 8r$$

7. Find the difference quotient of $f(x) = 3x^2$; that is find $\dfrac{f(x+h) - f(x)}{h}, h \neq 0$. (6 points)
Be sure to simplify.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{3(x+h)^2 - 3(x)^2}{h} \\ &= \frac{3(x^2 + 2xh + h^2) - 3x^2}{h} \\ &= \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} \\ &= \frac{6xh + 3h^2}{h} \\ &= 6x + 3h\end{aligned}$$

8. A line l_1 passes through the points: A(-4, -1) and B(6, 4). (16 points)

a. Write the equation of l_1 in slope-intercept form.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-1)}{6-(-4)} = \frac{4+1}{6+4} = \frac{5}{10} = \frac{1}{2}$$

$$y - y_1 = m_1(x - x_1)$$
$$y - 4 = \frac{1}{2}(x-6)$$
$$y - 4 = \frac{1}{2}x - 3$$
$$y = \frac{1}{2}x + 1$$

b. Find the equation of the line l_2 which is perpendicular to l_1 and has y-intercept: (0, 2).

$$m_2 = -\frac{1}{m_1} = -\frac{1}{(1/2)} = -2$$

$$y = m_2 x + b$$
$$y = -2x + 2$$

c. Graph both lines on the coordinate plane. (*Be sure to label each line*.)