Math 8: Exam 4 **Solutions**

You will not receive full credit if you do not clearly show how you are obtaining your answers. Show all work on this exam; do not attach other work. Circle your answers.

1. The price p (in dollars) and the quantity x sold of a certain product obey the demand equation $p = -\frac{1}{10}x + 150, \quad 0 \le x \le 1500$. What is the maximum revenue for this product? (10 points)

$$R(x) = xp = x\left(-\frac{1}{10}x + 150\right) = -\frac{1}{10}x^2 + 150x$$

<u>Vertex</u>

$$h = -\frac{b}{2a} = -\frac{150}{2(-1/10)} = \frac{150}{(1/5)} = 150(5) = 750$$

$$k = R(h) = R(750) = -\frac{1}{10}(750)^2 + 150(750) = -56{,}250 + 112{,}500 = 56{,}250$$

So the maximum possible revenue is \$56,250.

2. Form a third degree polynomial $f(x)$ with real coefficients and 4 and $2i$ as zeros. (5 points)

Since 2i is a zero and the coefficients of f are real, -2i must also be a zero.

$$\begin{aligned} f(x) &= (x-4)(x-2i)(x+2i) \\ &= (x-4)(x^2+4) \\ &= x^3 - 4x^2 + 4x - 16 \end{aligned}$$

3. Consider the polynomial function $f(x) = -x^4 + 4x^2$ (15 points)

a. Find the *y*-intercept of *f*(*x*).

$f(0) = 0 \quad (0,0)$

b. Find any *x*-intercepts of *f*(*x*), and determine whether the graph of *f* crosses or touches the *x*-axis at each *x*-intercept.

$$f(x) = 0$$
$$-x^4 + 4x^2 = 0$$
$$-x^2(x^2 - 4) = 0$$
$$-x^2(x-2)(x+2) = 0$$

$-x^2 = 0$ or $x - 2 = 0$ or $x + 2 = 0$

$x = 0$ $\quad$ $x = 2$ $\quad$ $x = -2$

0 has a multiplicity of 2 so the graph touches at (0, 0).
2 has a multiplicity of 1 so the graph crosses at (2, 0).
-2 has a multiplicity of 1 so the graph crosses at (-2, 0).

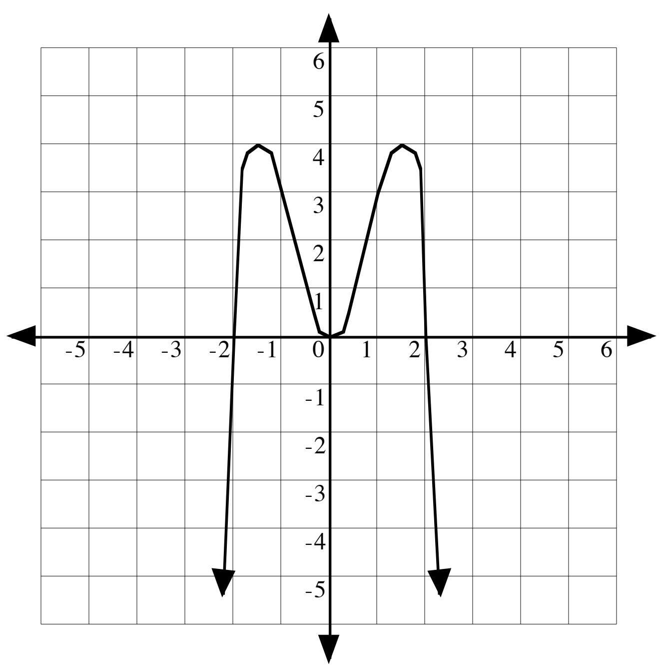
c. End behavior: find the power function that the graph of *f* resembles for large values of $|x|$.

$-x^4$

d. Determine the maximum number of turning points on the graph of *f*(*x*).

3

e. Sketch the graph of *f*(*x*) on the coordinate plane. *If necessary, find a few additional points on the graph.*



4. Find the horizontal or oblique asymptote of $R(x) = \dfrac{3x^4 + 4}{x^3 + 3x}$. Write *none* if there is none. (5 points)

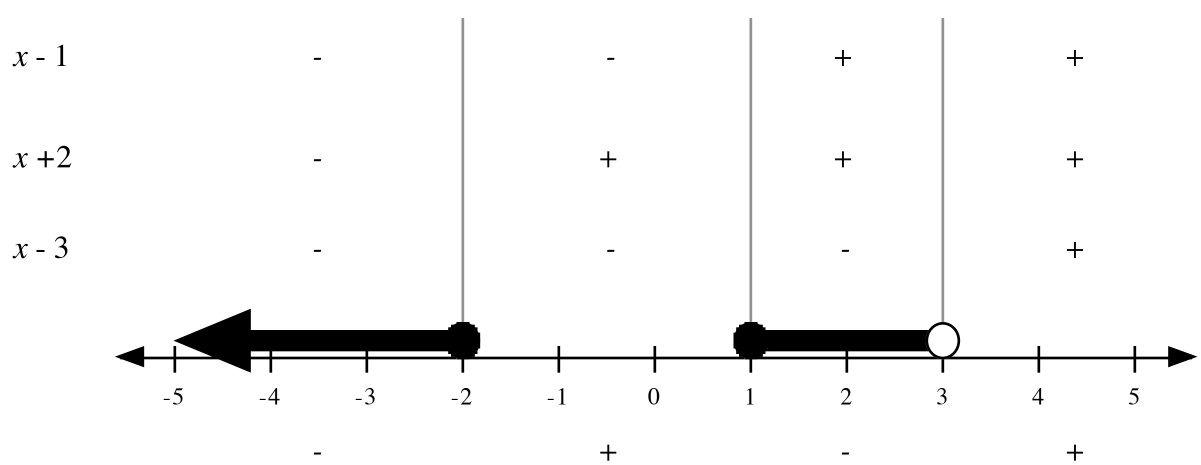
Since the degree of the numerator is one more than the degree of the denominator, there is an oblique asymptote.

$$\begin{array}{r|l} & \quad\quad\quad\quad\quad\quad\quad\quad 3x \\ \hline x^3 + 0x^2 + 3x + 0 & 3x^4 + 0x^3 + 0x^2 + 3x + 0 \\ & 3x^4 + 0x^3 + 9x^2 + 0x \\ \hline & \quad\quad\quad -9x^2 + 0x + 4 \end{array}$$

So $y = 3x$ is the oblique asymptote.

5. Solve and graph the solution set: $\dfrac{(x-1)(x+2)}{x-3} \le 0$. (10 points)
Write your answer in interval notation.

The function has x-intercepts at $(1, 0)$ and $(-2, 0)$, and a vertical asyptote at $x = 3$.



So the solution set is $(-\infty, -2] \cup [1, 3)$.

6. Let $f(x) = \dfrac{2x^2 + 8x}{x^2 + 3x - 4} = \dfrac{2x(x+4)}{(x-1)(x+4)} = \dfrac{2x}{x-1}$, where $x \neq 4$. (15 points)

a. Write the equations for any vertical asymptotes. If there aren't any then write *none*.

The simplified denominator is x - 1, so the vertical asymptote is $x = 1$.

b. Write the equation for the horizontal or oblique asymptote. If there isn't one then write *none*.

The numerator and the denominator have the same degree so the horizontal asymptote is the ratio of the leading coefficients: $x = 2$.

c. Find the coordinates of any "holes" in the function. If there aren't any then write *none*.

Since $x + 4$ is a factor of both the numerator and the denominator, there is a hole at $x = -4$. Substituting $x = -4$ into the reduced form of $f(x)$ produces $y = -1.6$.

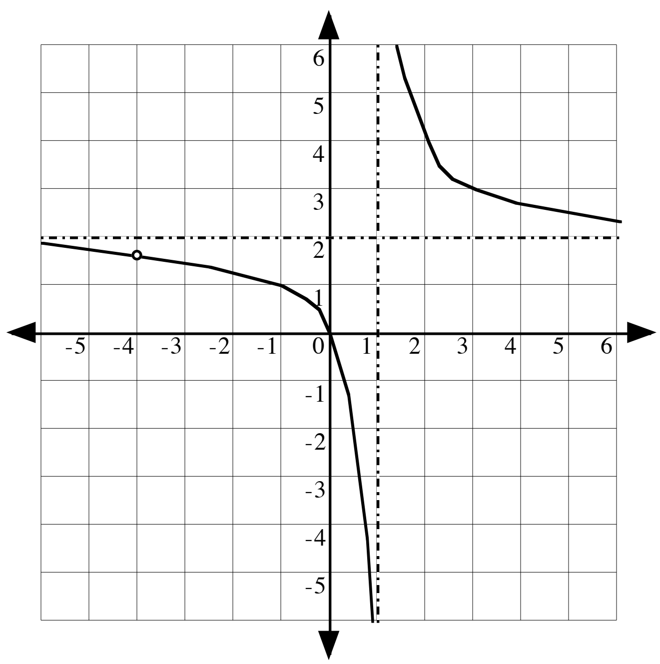
So there is a hole at (-4, 1.6).

d. Find all intercepts for the graph of $f(x)$.

Since $f(0) = 0$, the y-intercept is at (0, 0).

Since the numerator is $2x$, the only x-intercept is also at (0, 0).

e. Accurately graph $f(x)$. Be certain to draw all asymptotes, intercepts, and "holes." *Plot additional points on the graph if necessary.*



7. Consider the function $f(x) = 2x^3 + 13x^2 + 13x - 10$. (25 points)

a. Use the Rational Zeros Theorem to list all possible rational zeros of this function.

p: ±1, ±2, ±5, ±10
q: ±1, ±2

The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm 1, \pm 2, \pm\frac{5}{2}, \pm 5, \pm 10\right\}$.

b. Use Descartes' Rule of signs to determine the possible number of positive real zeros.

$f(x) =\ +\ +\ +\ -$

One change in sign means there is 1 positive real zero.

c. Use Descartes' Rule of signs to determine the possible number of negative real zeros.

$f(-x) =\ -\ +\ -\ -$

Two changes in sign means there are either 2 or 0 positive real zeros.

d. Find the zeros of this function.

−2	2	13	13	−10
		−4	−18	10
−5	2	9	−5	0
		−10	5	
1/2	2	−1	0	
		1		
	2	0		

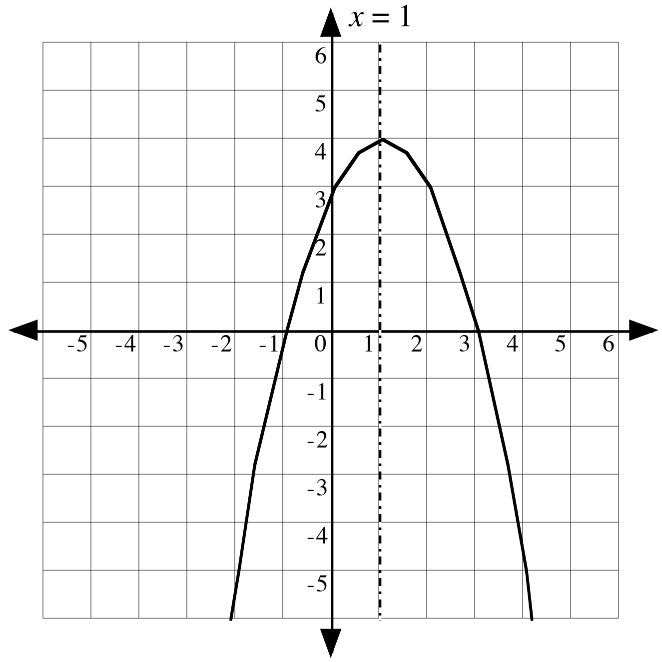
So the zeros are $\left\{-5, -2, \frac{1}{2}\right\}$.

e. Write the function as a product of linear factors.

$f(x) = 2(x+2)(x+5)\left(x - \frac{1}{2}\right)$ or $f(x) = (x+2)(x+5)(2x-1)$

8. Consider the function $f(x) = -x^2 + 2x + 3$. (15 points)

a. Find the coordinates of the vertex. Plot the vertex on the coordinate plane provided.



$$\begin{cases} a = -1 \\ b = 2 \\ c = 3 \end{cases}$$

$$h = -\frac{b}{2a} = -\frac{2}{2(-1)} = 1$$

$$f(h) = f(1) = -(1)^2 + 2(1) + 3 = 4$$

So the vertex is $(1, 4)$.

b. Find the equation of the axis of symmetry. Graph the axis on the coordinate plane.

$x = 1$

c. Find all x and y intercepts.

<u>x-intercepts (set $y = 0$)</u>

$$0 = -x^2 + 2x + 3$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x + 1 = 0$$

$$x = 3 \qquad\qquad x = -1$$

So the x-intercepts are $(3, 0)$ and $(-1, 0)$.

<u>y-intercept (set $x = 0$)</u>

$f(0) = 3$ so the y-intercept is $(0, -3)$.

d. Graph the function. *Plot and label at least three points in addition to the vertex and intercepts.*

e. State the domain and range of the function in set-builder notation.

$D{:}\left\{x \mid x \in \Re\right\}$ and $R{:}\left\{y \mid y \leq 4\right\}$.