Math 8: Exam 5 **Solutions**

You will not receive full credit if you do not clearly show how you are obtaining your answers. Show all work on this exam; do not attach other work. Circle your answers.

1. Write the equation $\log_6 \frac{1}{36} = -2$ in its equivalent exponential form. (3 points)

$6^{-2} = \frac{1}{36}$

2. Find the inverse of the one-to-one function $f(x) = -\frac{1}{2}x + 1$. (6 points)

$y = -\frac{1}{2}x + 1$

Switching *x* and *y* produces...

$$x = -\frac{1}{2}y + 1$$
$$-2x = y - 2$$
$$-2x + 2 = y$$

So $f^{-1}(x) = -2x + 2$

3. An initial deposit of \$5,000 grows at an annual rate of 8.5% for 5 years. (10 points)

a. Find the final balance resulting from annual compounding. $A = P(1+r)^t$

$$A = P(1+r)^t$$
$$= 5000(1+0.085)^5$$
$$= 5000(1.085)^5$$
$$\approx 5000(1.50366)$$
$$\approx \$7518.28$$

b. Find the final balance resulting from continuous compounding. $A = Pe^{rt}$

$$A = Pe^{rt}$$
$$= 5000e^{(0.085)(5)}$$
$$= 5000e^{0.425}$$
$$\approx 5000(1.52959)$$
$$\approx \$7647.95$$

4. Write the equation $\sqrt[3]{8} = 2$ in its equivalent logarithmic form. (3 points)

$$\sqrt[3]{8} = 2$$
$$8^{1/3} = 2$$
$$\log_8 2 = \frac{1}{3}$$

5. An artifact originally had 40 grams of carbon-14 present. The decay model $A = 40e^{-0.000121t}$ describes the amount of carbon-14 present, A, in grams after t years. Use this model to solve the problems below. (12 points)

a. How many grams of carbon-14 will be present after 19,030 years?

$$\begin{aligned} A &= 40e^{-0.000121t} \\ &= 40e^{-(0.000121)(19{,}030)} \\ &\approx 40e^{-2.30263} \\ &\approx 40(0.1) \\ &\approx 4 \end{aligned}$$

b. How old will the artifact be when only 20 grams of carbon-14 remain?

$$\begin{aligned} A &= 40e^{-0.000121t} \\ 20 &= 40e^{-0.000121t} \\ 0.5 &= e^{-0.000121t} \\ -0.000121t &= \ln(0.5) \\ t &= \frac{\ln(0.5)}{-0.000121} \\ t &\approx \frac{-0.69315}{-0.000121} \\ t &\approx 5728.5 \end{aligned}$$

6. Use properties of logarithms to expand each logarithmic expression as much as possible. (6 points)

a. $\log \sqrt[3]{\dfrac{x}{yz^2}} = \log(xy^{-1}z^2)^{1/3}$

$= \log(x^{1/3}y^{-1/3}z^{2/3})$

$= \dfrac{1}{3}\log x - \dfrac{1}{3}\log y + \dfrac{2}{3}\log z$

b. $\ln\left(\dfrac{x^3\sqrt{y}}{z}\right) = \ln(x^3y^{1/2}z^{-1})$

$= 3\ln x + \dfrac{1}{2}\ln y - \ln z$

8. $f(x) = 3x + 2$ and $g(x) = 2x - 3$ (8 points)

a. Find $(f \circ g)(x)$

$(f \circ g)(x) = f[g(x)]$

$= f(2x - 3)$

$= 3(2x - 3) + 2$

$= 6x - 9 + 2$

$= 6x - 7$

b. Find $(g \circ f)(x)$

$(g \circ f)(x) = g[f(x)]$

$= g(3x + 2)$

$= 2(3x + 2) - 3$

$= 6x + 4 - 3$

$= 6x + 1$

c. Are these functions inverses of each other?

No.

8. If $f(x) = 2^x + 1$ then its inverse is $f^{-1}(x) = \log_2(x-1)$. (20 points)

a. Find the coordinates of at least four points for each function. Plot $f(x)$ and $f^{-1}(x)$ on the coordinate plane provided. Be sure to graph the asymptote for each function.

x	$f(x)$
2	5
1	3
0	2
-1	1.5
-2	1.25
-3	1.125
-4	1.0625
-5	1.03125
-6	1.015625

x	$f^{-1}(x)$
5	2
3	1
2	0
1.5	-1
1.25	-2
1.125	-3
1.0625	-4
1.03125	-5
1.015625	-6

b. Write the domain and range of $f(x)$ in interval notation.

$D: (-\infty, \infty)$

$R: (1, \infty)$

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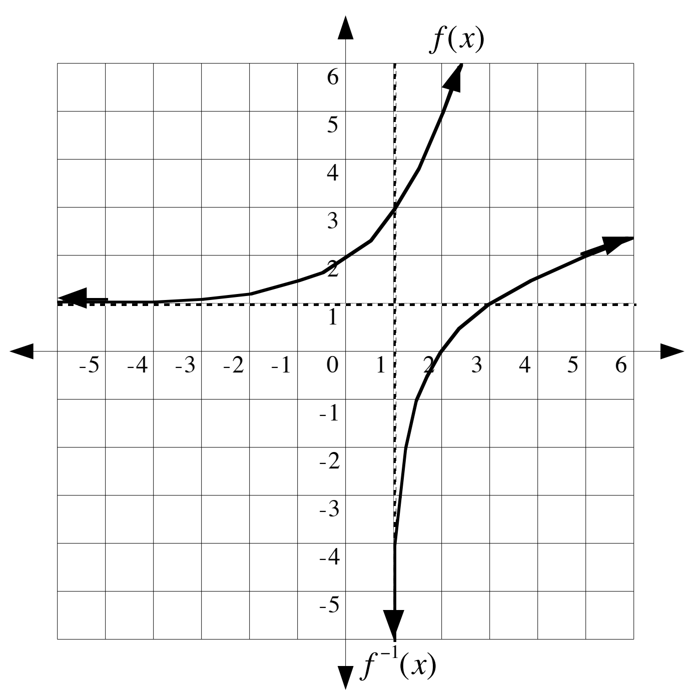

9. Solve each logarithmic equation. (6 points)

a.

$$x = \log_6 \sqrt{6}$$
$$x = \log_6 (6^{1/2})$$
$$x = \frac{1}{2}$$

b.

$$\log_7 x = -2$$
$$x = 7^{-2}$$
$$x = \frac{1}{49}$$

10. Solve the exponential equation $2^x = 3^{x-1}$. Round the solution to two decimal places. (10 points)

$$2^x = 3^{x-1}$$
$$\ln(2^x) = \ln(3^{x-1})$$
$$x\ln 2 = (x-1)\ln 3$$
$$x\ln 2 = x\ln 3 - \ln 3$$
$$x\ln 2 - x\ln 3 = -\ln 3$$
$$x(\ln 2 - \ln 3) = -\ln 3$$
$$x = \frac{-\ln 3}{\ln 2 - \ln 3}$$
$$x \approx 2.71$$

11. Use properties of logarithms to condense each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. (6 points)

a. $\log x + \frac{1}{2}\log y - 4\log z = \log x + \log y^{1/2} + \log z^{-4}$

$$= \log(xy^{1/2}z^{-4})$$

$$= \log\left(\frac{x\sqrt{y}}{z^4}\right)$$

b. $2\ln x - (3\ln y + \ln z) = 2\ln x - 3\ln y - \ln z$

$$= \ln x^2 + \ln y^{-3} + \ln z^{-1}$$

$$= \ln(x^2 y^{-3} z^{-1})$$

$$= \ln\left(\frac{x^2}{y^3 z}\right)$$

12. Solve the logarithmic equation $\log_4 x + \log_4(3x + 8) = 2$. Be sure to reject any value of x that produces the logarithm of a negative number or the logarithm of 0. (10 points)

$$\log_4 x + \log_4(3x + 8) = 2$$

$$\log_4[x(3x + 8)] = 2$$

$$x(3x + 8) = 4^2$$

$$3x^2 + 8x = 16$$

$$3x^2 + 8x - 16 = 0$$

$$(3x - 4)(x + 4) = 0$$

$3x - 4 = 0$ or $x + 4 = 0$

$3x = 4$ $\qquad x = -4$

$x = \frac{4}{3}$

Since $x = -4$ produces the logarithm of a negative number, the only solution is $x = \frac{4}{3}$.