

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. a. Solve the inequality  $-1 < \frac{2x-3}{7} \leq \frac{3}{7}$ . (10 points)

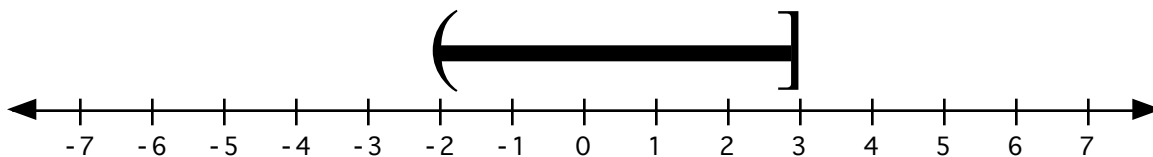
$$\begin{aligned} -1 < \frac{2x-3}{7} \leq \frac{3}{7} \\ -7 < 2x-3 \leq 3 \\ -4 < 2x \leq 6 \\ -2 < x \leq 3 \end{aligned}$$

- b. Express your answer using set notation.      c. Express your answer using interval notation.

$$\{x \mid -2 < x \leq 3\}$$

$$(-2, 3]$$

- d. Graph the solution set.



2. Write the expression,  $\left(\frac{3}{5} - \frac{1}{5}i\right)^2$  in the standard form  $a + bi$ . (6 points)

$$\left(\frac{3}{5} - \frac{1}{5}i\right)^2 = \left(\frac{3}{5}\right)^2 - 2\left(\frac{3}{5}\right)\left(\frac{1}{5}i\right) + \left(\frac{1}{5}i\right)^2 = \frac{9}{25} - \frac{6}{25}i + \frac{1}{25}i^2 = \frac{9}{25} - \frac{6}{25}i - \frac{1}{25} = \frac{8}{25} - \frac{6}{25}i$$

3. A coffee manufacturer wants to market a new blend of coffee that sells for \$3.90 per pound by mixing two coffees that sell for \$2.75 and \$5 per pound, respectively. What amounts of each coffee should be blended to obtain 100 pounds of the desired mixture? (12 points)

Let  $x$  = the weight of the \$2.75/lb coffee needed.

	Quantity	Unit Price	Value
cheap	$x$	2.75	$2.75x$
expensive	$100 - x$	5	$5(100 - x)$
blend	100	3.9	390

$$\begin{aligned}
 2.75x + 5(100 - x) &= 390 \\
 275x + 500(100 - x) &= 39,000 \\
 275x + 50,000 - 500x &= 39,000 \\
 -225x + 50,000 &= 39,000 \\
 -225x &= -11,000 \\
 x &= 48\frac{8}{9}
 \end{aligned}$$

$$100 - x = 100 - 48\frac{8}{9} = 51\frac{1}{9}$$

We need  $48\frac{8}{9}$  pounds of the \$2.75/lb coffee and  $51\frac{1}{9}$  pounds of the \$5/lb coffee.

4. Write the expression,  $\frac{7}{2-3i}$  in the standard form  $a + bi$ . (6 points)

$$\frac{7}{(2-3i)} \cdot \frac{(2+3i)}{(2+3i)} = \frac{7(2+3i)}{(2)^2 + (3)^2} = \frac{14+21i}{4+9} = \frac{14+21i}{13} = \frac{14}{13} + \frac{21}{13}i$$

5. Find the real solutions of the equation,  $\sqrt{3x-5} - \sqrt{x+7} = 2$ . (12 points)

$$\begin{aligned} \sqrt{3x-5} - \sqrt{x+7} &= 2 \\ \sqrt{3x-5} &= 2 + \sqrt{x+7} \\ (\sqrt{3x-5})^2 &= (2 + \sqrt{x+7})^2 \\ 3x - 5 &= 4 + 4\sqrt{x+7} + (x+7) \\ 3x - 5 &= x + 11 + 4\sqrt{x+7} \\ 2x - 16 &= 4\sqrt{x+7} \\ x - 8 &= 2\sqrt{x+7} \\ (x-8)^2 &= (2\sqrt{x+7})^2 \\ x^2 - 16x + 64 &= 4(x+7) \\ x^2 - 16x + 64 &= 4x + 28 \\ x^2 - 20x + 36 &= 0 \\ (x-18)(x-2) &= 0 \\ x-18=0 \quad \text{or} \quad x-2=0 \\ x=18 \quad \quad \quad x=2 \end{aligned}$$

Checking shows that 2 is not a solution, so 18 is the only solution.

6. Find the real solutions, if any, of the equation,  $6x^2 + 14x = 9$ . Use the quadratic formula. (12 points)

$$\begin{aligned} 6x^2 + 14x &= 9 \\ 6x^2 + 14x - 9 &= 0 \\ \begin{cases} a = 6 \\ b = 14 \\ c = -9 \end{cases} & \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ & \quad x = \frac{-(14) \pm \sqrt{(14)^2 - 4(6)(-9)}}{2(6)} \\ & \quad x = \frac{-14 \pm \sqrt{412}}{12} \\ & \quad x = \frac{-14 \pm 2\sqrt{103}}{12} \\ & \quad x = \frac{-7 \pm \sqrt{103}}{6} \end{aligned}$$

7. a. Solve the inequality,  $-|3-2x| < -7$ . (10 points)

$$\begin{aligned} -|3-2x| &< -7 \\ |3-2x| &> 7 \end{aligned}$$

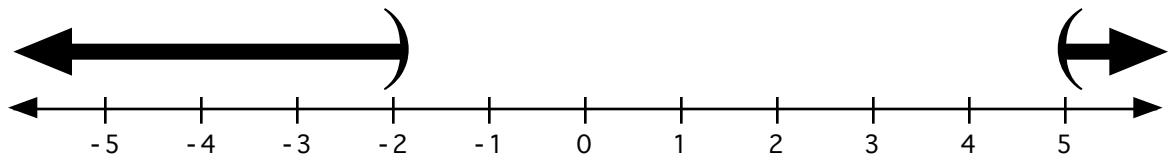
$$\begin{aligned} 3-2x > 7 & \quad \text{or} \quad 3-2x < -7 \\ -2x > 4 & \quad \quad \quad -2x < -10 \\ x < -2 & \quad \quad \quad x > 5 \end{aligned}$$

- b. Express your answer using set notation.      c. Express your answer using interval notation.

$$\{x \mid x < -2 \text{ or } x > 5\}$$

$$(-\infty, -2) \cup (5, \infty)$$

- d. Graph the solution set.



8. Solve the equation,  $\frac{x}{x^2-9} + \frac{4}{x+3} = \frac{3}{x^2-9}$ . (10 points)

$$\begin{aligned} \frac{x}{(x+3)(x-3)} + \frac{4}{x+3} &= \frac{3}{(x+3)(x-3)} \\ (x+3)(x-3) \left( \frac{x}{(x+3)(x-3)} + \frac{4}{x+3} \right) &= \left( \frac{3}{(x+3)(x-3)} \right) (x+3)(x-3) \\ x + 4(x-3) &= 3 \\ x + 4x - 12 &= 3 \\ 5x - 12 &= 3 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

Checking reveals that 3 is not a solution. Thus there are no solutions.

9. Find the real solutions of the equation,  $2x^{2/3} - 5x^{1/3} - 3 = 0$ .

(10 points)

$$2x^{2/3} - 5x^{1/3} - 3 = 0$$

$$2u^2 - 5u - 3 = 0 \quad \text{Let } u = x^{1/3}$$

$$(2u+1)(u-3) = 0$$

$$2u+1=0 \quad \text{or} \quad u-3=0$$

$$2u=-1 \quad u=3$$

$$u=-\frac{1}{2} \quad x^{1/3}=3$$

$$x^{1/3}=-\frac{1}{2} \quad (x^{1/3})^3=(3)^3$$

$$x=27$$

$$(x^{1/3})^3 = \left(-\frac{1}{2}\right)^3$$

$$x = -\frac{1}{8}$$

10. Solve the equation,  $2x^2 + 6x + 1 = 0$ , by completing the square.

(12 points)

$$2x^2 + 6x = -1$$

$$x^2 + 3x = -\frac{1}{2}$$

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = -\frac{1}{2} + \left(\frac{3}{2}\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 = -\frac{2}{4} + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{7}{4}$$

$$\sqrt{\left(x + \frac{3}{2}\right)^2} = \pm \sqrt{\frac{7}{4}}$$

$$x + \frac{3}{2} = \pm \frac{\sqrt{7}}{2}$$

$$x = -\frac{3}{2} \pm \frac{\sqrt{7}}{2}$$

$$x = \frac{-3 \pm \sqrt{7}}{2}$$