Math 8: Exam 3 (Fall 2009) **Solutions**

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. Let $f(x) = \begin{cases} \frac{1}{2}x + 4 & \text{if } -4 \le x < 2 \\ (x-3)^2 & \text{if } 2 \le x < 5 \end{cases}$ (14 points)

a. Evaluate $f(0)$

$$f(0) = \frac{1}{2}(0) + 4 = 4$$

b. Evaluate $f(2)$

$$f(2) = (2-3)^2 = (-1)^2 = 1$$

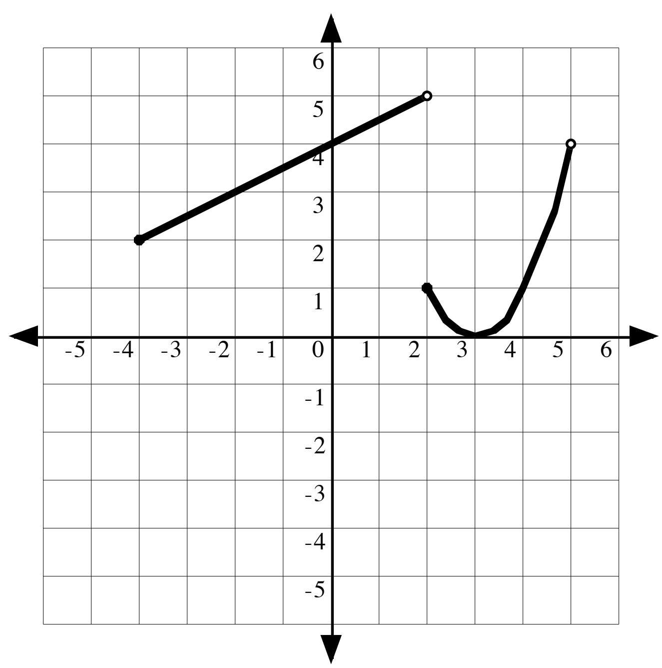
c. Evaluate $f(-4)$

$$f(-4) = \frac{1}{2}(-4) + 4 = -2 + 4 = 2$$

d. Evaluate $f(3)$

$$f(3) = (3-3)^2 = (0)^2 = 0$$

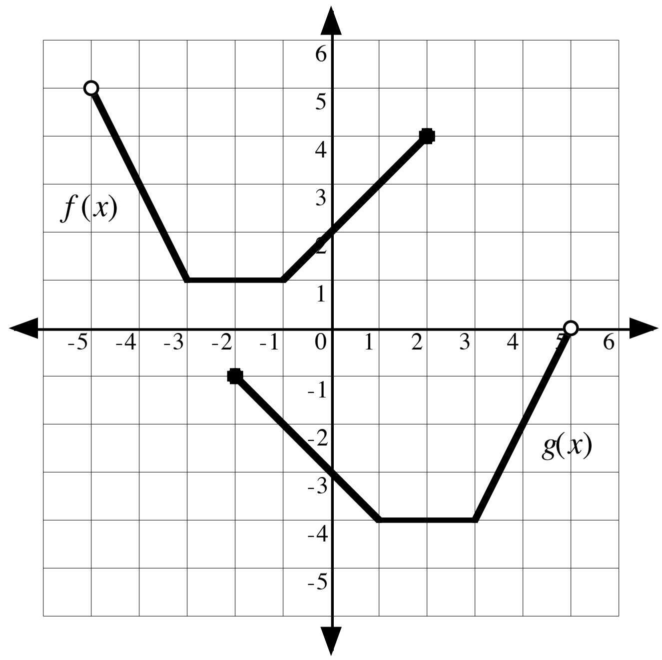
e. Graph $f(x)$ on the coordinate plane provided.



2. Given the graph of $f(x)$ shown below, graph $g(x) = f(-x) - 5$. (10 points)
Explain in words how the graph of $g(x)$ is obtained from the graph of $f(x)$.

Subtracting 5 from the dependent variable, $f(x)$, translates the graph five units down.

Multiplying the independent variable, x, by -1 reflects the graph about the y-axis.



3. Let $h(x) = 2x^2 + 4$. (10 points)

a. Find the average rate of change from 2 to 3.

$$\frac{h(3)-h(2)}{3-2} = \frac{[2(3)^2+4]-[2(2)^2+4]}{1} = \frac{22-12}{1} = \frac{10}{1} = 10$$

b. Find the equation of the secant line containing $(2, h(2))$ and $(3, h(3))$.

$m = 10$

$(x_1, y_1) = (2, h(2)) = (2, 12)$

$$y - y_1 = m(x - x_1)$$
$$y - 12 = 10(x - 2)$$
$$y - 12 = 10x - 20$$
$$y = 10x - 8$$

c. Determine whether $h(x)$ is *even*, *odd*, or *neither*.

$h(-x) = 2(-x)^2 + 4 = 2x^2 + 4 = h(x)$

So $h(x)$ is *even*.

4. Let $y = \dfrac{x^2 - 4}{x}$. (15 points)

a. List the x-intercepts of the relation. If there are no x-intercepts then write *no x-intercepts*.

Set $y = 0$.

$$\begin{aligned} 0 &= \frac{x^2 - 4}{x} \\ 0 &= x^2 - 4 \\ 4 &= x^2 \\ \pm 2 &= x \end{aligned}$$

So the x-intercepts are $(2,0), (-2,0)$.

b. List the y-intercepts of the relation. If there are no y-intercepts then write *no y-intercepts*.

Set $x = 0$ $\quad y = \dfrac{(0)^2 - 4}{2(0)} = \dfrac{-4}{0}$ which is undefined

So there are *no y-intercepts*.

c. Test the relation for each of the symmetries listed below, then state your conclusions.

<u>x-axis symmetry</u>

(change y into -y)

$$\begin{aligned} -y &= \frac{x^2 - 4}{2x} \\ y &= \frac{4 - x^2}{2x} \end{aligned}$$

<u>y-axis symmetry</u>

(change x into -x)

$$\begin{aligned} y &= \frac{(-x)^2 - 4}{2(-x)} \\ y &= \frac{x^2 - 4}{-2x} \\ y &= -\frac{x^2 - 4}{2x} \end{aligned}$$

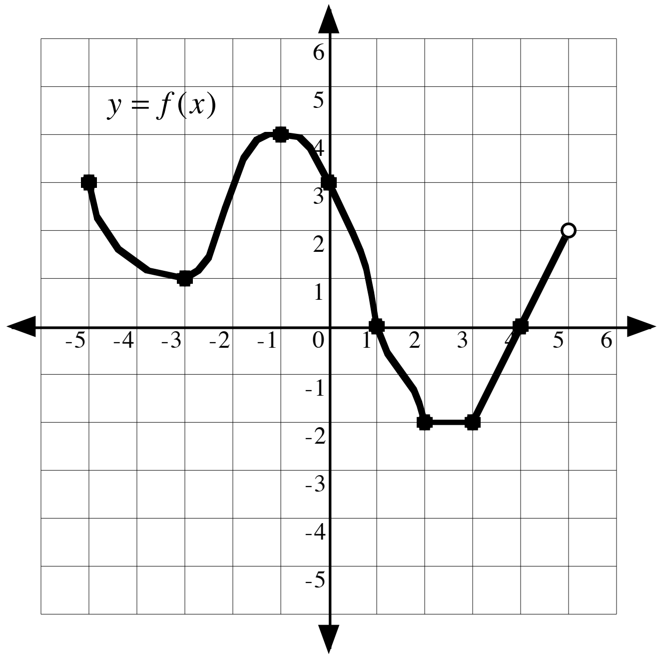
<u>origin symmetry</u>

(change x into -x and y into -y)

$$\begin{aligned} -y &= \frac{(-x)^2 - 4}{2(-x)} \\ -y &= -\frac{x^2 - 4}{2x} \\ y &= \frac{x^2 - 4}{2x} \end{aligned}$$

So the relation is symmetric about the origin.

5. Use the graph of $f(x)$ shown below to determine the indicated characteristics. (20 points)



a. Determine the function's domain.

$[-5, 5)$

b. Determine the function's range.

$[-2, 4]$

c. Determine the function's *x*-intercepts, if any.

$(1, 0), (4, 0)$

d. Determine the function's *y*-intercept, if any.

$(0, 3)$

e. Determine the intervals on which the function is increasing, if any.

$(-3, -1) \cup (3, 5)$

f. Determine the intervals on which the function is decreasing, if any.

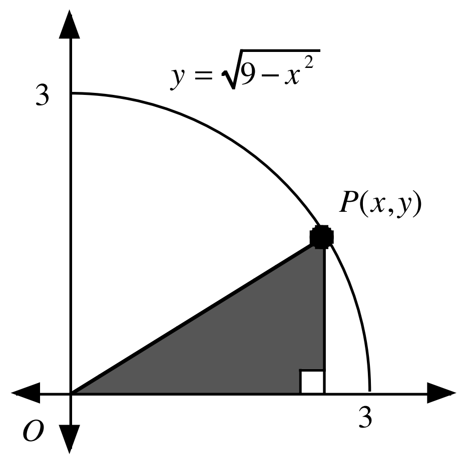
$(-5, -3) \cup (-1, 2)$

g. Determine the intervals on which the function is constant, if any.

$(2, 3)$

6. A right triangle is inscribed in the first quadrant of the semicircle $y = \sqrt{9 - x^2}$ so that the right-angle vertex is on the x-axis, another vertex is at the origin $(0, 0)$, and the third vertex is on the circle at point $P(x, y)$. Express the area of the triangle as a function of x. (8 points)

$$A = \frac{1}{2}bh = \frac{1}{2}xy = \frac{1}{2}x\sqrt{9 - x^2}$$



7. Find the difference quotient of $f(x) = x^2 - 8$; that is find $\dfrac{f(x+h) - f(x)}{h}, h \neq 0$. (7 points)

Be sure to simplify your answer.

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{[(x+h)^2 - 8] - [x^2 - 8]}{h} \\
&= \frac{x^2 + 2xh + h^2 - 8 - x^2 + 8}{h} \\
&= \frac{2xh + h^2}{h} \\
&= \frac{h(2x + h)}{h} \\
&= 2x + h
\end{aligned}$$

8. A line l_1 passes through the points: A(-6, -2) and B(6, 6). (16 points)

a. Write the equation of l_1 in slope-intercept form.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-2)}{6-(-6)} = \frac{6+2}{6+6} = \frac{8}{12} = \frac{2}{3}$$

$$y - y_1 = m_1(x - x_1)$$
$$y - 6 = \frac{2}{3}(x-6)$$
$$y - 6 = \frac{2}{3}x - 4$$
$$y = \frac{2}{3}x + 2$$

b. Find the equation of the line l_2 which is parallel to l_1 and has *y*-intercept: (0, -1).

$$m_2 = m_1 = \frac{2}{3}$$

$$y = m_2 x + b$$
$$y = \frac{2}{3}x - 1$$

c. Graph both lines on the coordinate plane. (*Be sure to label each line*.)