

# Algebra Tiles

Algebra tiles can be used to physically represent the addition, subtraction and multiplication of polynomials. The tiles use area to represent the quantities  $1, x$  and  $x^2$ . The small square tiles have sides of unit length, and thus have an area of 1 square unit. The larger rectangular tiles have sides of length 1 and  $x$ , and so have an area of  $x$  square units. The large square tiles have sides of length  $x$ , and therefore have an area of  $x^2$  square units.

The opposite quantities  $-1, -x$  and  $-x^2$  are represented by using the reverse sides of the tiles which will be a different color. A neutral pair consisting of a tile and its opposite has a value of zero.



Note: The length of the sides are designed so that the length  $x$  is not an integer multiple of the length 1. This means that you can not create an  $x$  tile out of 1 tiles and you can not create an  $x^2$  tile out of either 1 or  $x$  tiles. This reinforces the idea that  $x$  as an unknown quantity.

## Adding Polynomials

Put down the tiles which represent the first polynomial. Do the same for the second polynomial. Remove any neutral pairs which have been created. The remaining tiles represent the answer.

Example:  $(x + 2) + (x^2 - 3x - 1)$

Put down one  $x$  tile and two 1 tiles to represent the first polynomial. Now put down one  $x^2$  tile, three  $-x$  tiles and one  $-1$  tile. Remove the neutral pairs. One  $x^2$  tile, two  $-x$  tiles, and one 1 tile remain so the answer is  $x^2 - 2x + 1$



Note: The removal of neutral pairs and the grouping of similar tiles illustrates the process of combining like terms.

## Subtracting Polynomials

Put down the tiles which represent the first polynomial. Do the same for the second polynomial. Now turn over all the tiles from the second polynomial, thus reversing their signs. Remove any neutral pairs which have been created. The remaining tiles represent the answer.

Example:  $(x + 2) - (x^2 - 3x - 1)$

Put down one  $x$  tile and two 1 tiles to represent the first polynomial. Now put down one  $x^2$  tile, three  $-x$  tiles and one  $-1$  tile. Turn over the tiles from the second polynomial. Remove the neutral pairs. One  $-x^2$  tile, four  $x$  tiles, and three 1 tiles remain so the answer is  $-x^2 + 4x + 3$



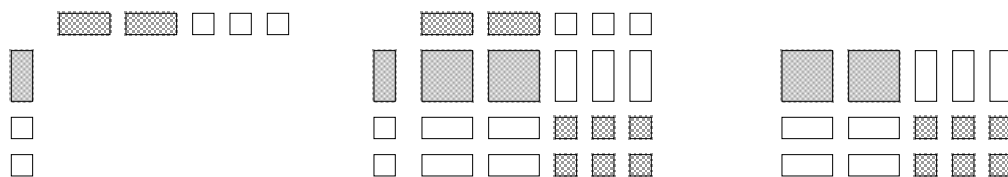
Note: By reversing the sign of the second polynomial we turn the subtraction problem into an addition problem. This concept is often difficult to grasp when solving problems on paper, but the tiles make it very intuitive.

## Multiplying Polynomials

Make a vertical line from the tiles representing the first polynomial. Line up the tiles for the second polynomial at right angles to the first. Fill in the rectangular area which you have created. If a section is created by two tiles with the same color, fill it with a dark tile. If a section is created by two tiles with different colors, fill it with a white tile. Remove the tiles from the original polynomials. Remove any neutral pairs. The remaining tiles represent the answer.

Example:  $(x - 2)(2x - 3)$

Line up the tiles representing  $x - 2$  vertically. Line up the tiles for  $2x - 3$  horizontally. Fill the rectangular areas. Remove the original polynomials. There are no neutral pairs in this case. Two  $x^2$  tiles, seven  $-x$  tiles, and six  $1$  tiles remain so the answer is  $2x^2 - 7x + 6$



Notes: The example above demonstrates the FOIL method for multiplying binomials.

You can illustrate the distributive property by multiplying a monomial by a binomial.

## Practice Problems

Use the problems below to practice working with algebra tiles.

1.  $(3x + 2) + (x + 4)$
2.  $(x - 2) + (x + 3)$
3.  $(2 - x) + (x - 2)$
4.  $(x^2 + x + 3) + (3x + 1)$
5.  $(-x^2 - x + 1) + (x^2 + 2x - 3)$
6.  $(-2x^2 - 3x + 1) + (x^2 + 2x)$
7.  $(2x^2 - x + 1) + (x^2 + 2x - 3)$
8.  $(2x^2 - 3x - 1) + (-x^2 - 3)$
9.  $(2x + 3) - (x + 3)$
10.  $(x - 3) - (x + 2)$
11.  $(x - 2) - (2 - x)$
12.  $(3x + 2) - (2x^2 + x + 1)$
13.  $(-x^2 - 2x + 1) - (x^2 + x - 3)$
14.  $(x^2 - 2x) - (-2x^2 - 5x + 1)$
15.  $(2x^2 - x + 1) - (x^2 + 2x - 3)$
16.  $(2x^2 - x - 3) - (-x^2 - 2)$
17.  $2x(x + 1)$
18.  $(-x)(3x - 2)$
19.  $(x + 2)(2x + 1)$
20.  $(2x - 2)(x + 3)$
21.  $(2x + 1)(2x + 1)$
22.  $(2x + 1)(2x - 1)$
23.  $(2 - x)(x - 1)$
24.  $(x + 2)(1 - 2x)$

## Solutions

1.  $4x + 6$
2.  $2x + 1$
3.  $0$
4.  $x^2 + 4x + 4$
5.  $x - 2$
6.  $-x^2 - x + 1$
7.  $3x^2 + x - 2$
8.  $x^2 - 3x - 4$
9.  $x$
10.  $-5$
11.  $2x - 4$
12.  $-2x^2 + 2x + 1$
13.  $-2x^2 - 3x + 4$
14.  $3x^2 + 3x - 1$
15.  $x^2 - 3x + 4$
16.  $3x^2 - x - 1$
17.  $2x^2 + 2x$
18.  $-3x^2 + 2x$
19.  $2x^2 + 5x + 2$
20.  $2x^2 + 4x - 6$
21.  $4x^2 + 4x + 1$
22.  $4x^2 - 1$
23.  $-x^2 + 3x - 2$
24.  $-2x^2 - 3x + 2$