

Casting Out Nines

Casting out nines is a technique which can be used as a divisibility test, to check complicated arithmetic operations and which has many practical applications.

Methods

There are several methods for casting out nines. As an example, we will use the number 3,267,983.

- 1.) Sum all of the digits of the number. Of the result has more than one digit, then sum all of its digits. Continue until you have a single digit.

Summing the digits of 3,267,983 produces: $3 + 2 + 6 + 7 + 9 + 8 + 3 = 38$

Summing the digits of 38 produces: $3 + 8 = 11$

Summing the digits of 11 produces: $1 + 1 = 2$

So the result is 2.

- 2.) Working from left to right, begin to add the digits. If the sum becomes greater than or equal to nine, then subtract nine from the total and continue adding digits.

The sum of 3 and 2 is 5. ~~3~~ ~~2~~ 6 7 9 8 3
5

The sum of 5 and 6 is 11. Subtracting 9 produces 2. ~~3~~ ~~2~~ ~~6~~ 7 9 8 3
~~5~~ 2

The sum of 2 and 7 is 9. Subtracting 9 produces 0. ~~3~~ ~~2~~ ~~6~~ ~~7~~ 9 8 3
~~5~~ ~~2~~ 0

The sum of 0 and 9 is 9. Subtracting 9 produces 0. ~~3~~ ~~2~~ ~~6~~ ~~7~~ ~~9~~ 8 3
~~5~~ ~~2~~ ~~0~~ 0

The sum of 0 and 8 is 8. ~~3~~ ~~2~~ ~~6~~ ~~7~~ ~~9~~ ~~8~~ 3
~~5~~ ~~2~~ ~~0~~ ~~0~~ 8

The sum of 8 and 3 is 11. Subtracting 9 produces 2. ~~3~~ ~~2~~ ~~6~~ ~~7~~ ~~9~~ ~~8~~ ~~3~~
~~5~~ ~~2~~ ~~0~~ ~~0~~ ~~8~~ 2

So the result is 2.

- 3.) Eliminate any combinations of digits which sum to nine. Apply method 1 or 2 to the remaining digits.

The digits are 3 2 6 7 9 8 3

3 and 6 sum to nine so we eliminate them: ~~3~~ 2 ~~6~~ 7 9 8 3

2 and 7 sum to nine so we eliminate them: ~~3~~ ~~2~~ ~~6~~ ~~7~~ 9 8 3

9 is nine so we eliminate it: ~~3~~ ~~2~~ ~~6~~ ~~7~~ ~~9~~ 8 3

Summing the remaining digits produces: $8 + 3 = 11$

Summing the digits of 11 produces: $1 + 1 = 2$

So the result is 2.

Note: Casting out nines is the same thing as using the modulo 9 mathematical system where numbers larger than 9 “wrap around.” Thus 9 is the same as zero and 10 is the same as 1. Clocks use a similar system (modulo 12) where 1 follows 12.

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