

Integer Tiles

Integer tiles can be used to physically represent the addition, subtraction and multiplication of integers. Positive tiles have a value of +1 and negative tiles have a value of -1. A neutral pair consisting of one positive tile and one negative tile has a value of zero. A neutral field consists of many neutral pairs.

Addition

Put down the tiles (positive or negative) which represent the first number. Do the same for the second number. Remove any neutral pairs which have been created. The remaining tiles represent the answer.

Example

$$2 + 3 = ?$$

Procedure

Put down two positive tiles, and then put down three positive tiles. Five positive tiles remain so the answer is 5.

$$(-2) + (-1) = ?$$

Put down two negative tiles, and then put down one negative tile. Three negative tiles remain so the answer is -3.

$$2 + (-3) = ?$$

Put down two positive tiles, and then put down three negative tiles. Remove the two neutral pairs. One negative tile remains so the answer is -1.

$$(-2) + 4 = ?$$

Put down two negative tiles, and then put down four positive tiles. Remove the two neutral pairs. Two positive tiles remain so the answer is 2.

Note:

The removal of neutral pairs is a demonstration of the property of cancellation.

Subtraction

Start with a neutral field. Put down the tiles (positive or negative) which represent the first number. Then take away the tiles which represent the second number. Remove any neutral pairs. The remaining tiles represent the answer.

Example

$$3 - 1 = ?$$

Procedure

Start with a neutral field. Put down three positive tiles, and then take away one positive tile. Remove any neutral pairs. Two positive tiles remain so the answer is 2.

$$2 - 4 = ?$$

Start with a neutral field. Put down two positive tiles, and then take away four positive tiles. Remove any neutral pairs. Two negative tiles remain so the answer is -2.

$$(-2) - (-1) = ?$$

Start with a neutral field. Put down two negative tiles, and then take away one negative tile. Remove any neutral pairs. One negative tile remains so the answer is -1.

$$(-2) - (-3) = ?$$

Start with a neutral field. Put down two negative tiles, and then take away three negative tiles. Remove any neutral pairs. One positive tile remains so the answer is 1.

$$2 - (-3) = ?$$

Start with a neutral field. Put down two positive tiles, and then take away three negative tiles. Remove any neutral pairs. Five positive tiles remain so the answer is 5.

$$(-2) - 4 = ?$$

Start with a neutral field. Put down two negative tiles, and then take away four positive tiles. Remove any neutral pairs. Six negative tiles remain so the answer is -6.

Note:

Once this process has been practiced, it is a simple matter to show that it is equivalent to changing the sign of the second number and then adding the two numbers.

Multiplication

Start with a neutral field. You will add or take away groups of tiles, then remove any neutral pairs. The remaining tiles represent the answer. The second number indicates the number and type of tiles in each group. The first number indicates the number of groups, and the sign of the first number indicates whether the groups should be added or taken away.

Example

$(3)(2) = ?$

Procedure

Start with a neutral field. Put in three groups of two positive tiles each. Remove any neutral pairs. Six positive tiles remain so the answer is 6.

$(4)(-2) = ?$

Start with a neutral field. Put in four groups of two negative tiles each. Remove any neutral pairs. Eight negative tiles remain so the answer is -8.

$(-2)(3) = ?$

Start with a neutral field. Take away two groups of three positive tiles each. Remove any neutral pairs. Six negative tiles remain so the answer is -6.

$(-3)(-4) = ?$

Start with a neutral field. Take away three groups of four negative tiles each. Remove any neutral pairs. Twelve positive tiles remain so the answer is 12.

Note:

Exchanging the roles of the two number produces the same result, thus demonstrating the commutative property of multiplication.

Practice Problems

Use the problems below to practice working with integer tiles.

Addition

1. $2 + 4$

2. $(-2) + 5$

3. $(-1) + (-4)$

4. $2 + (-5)$

5. $(-3) + (-2)$

6. $4 + (-2)$

7. $3 + 4$

8. $(-4) + 3$

9. $(-7) + 4$

10. $8 + 5$

11. $6 + (-2)$

12. $(-7) + (-3)$

Subtraction

13. $5 - 2$

14. $1 - 3$

15. $(-3) - (-2)$

16. $(-3) - (-4)$

17. $4 - (-2)$

18. $(-3) - 3$

19. $6 - 5$

20. $4 - 7$

21. $(-5) - (-3)$

22. $(-1) - (-5)$

23. $6 - (-3)$

24. $(-5) - 2$

Multiplication

25. $(2)(3)$

26. $(-2)(2)$

27. $(3)(-4)$

28. $(-1)(-4)$

29. $(-1)(3)$

30. $(4)(3)$

31. $(-3)(-2)$

32. $(5)(-1)$

33. $(3)(-3)$

34. $(-4)(-2)$

35. $(2)(2)$

36. $(-4)(2)$

Solutions

1. 6

2. 3

3. -5

4. -3

5. -5

6. 2

7. 7

8. -1

9. -3

10. 13

11. 4

12. -10

13. 3

14. -2

15. -1

16. 1

17. 6

18. -6

19. 1

20. -3

21. -2

22. 4

23. 9

24. -7

25. 6

26. -4

27. -12

28. 4

29. -3

30. 12

31. 6

32. -5

33. -9

34. 8

35. 4

36. -8

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