Lab - Kepler's Laws

Purpose: plot a planetary orbit and apply Kepler's Laws.

Materials: polar graph paper, pencil, metric ruler

Procedure:

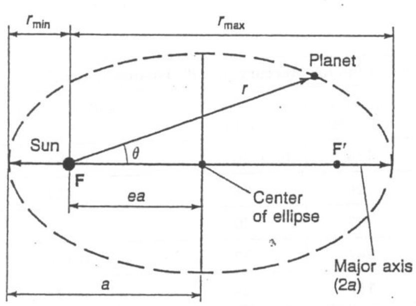
1. Orient your polar graph paper so that the zero degree point is on your right as you view the graph paper. The sun is located at the center of the paper. Label the sun without covering the center mark. Move about the center in a counter-clockwise direction as you measure and mark the longitude.
2. Select an appropriate scale to represent the values for the radius vectors of Mercury's positions. Since Mercury is closer to the sun than Earth, the value of the radius vector will always be less than 1 AU. In this step, then, each concentric circle could represent one-tenth of an AU.
3. Table 1 provides the heliocentric positions of Mercury over a period of several months. Select the set of data for October 1 and locate the given longitude on the polar graph paper. Measure out along the longitude line an appropriate distance, in your scale, for the radius vector for this date. Make a small dot at this point to represent Mercury's distance from the sun. Write the date next to this point.
4. Repeat the procedure, plotting all given longitudes and associated radius vectors.
5. After plotting all the data, carefully connect the points of Mercury's positions and sketch the orbit of Mercury.

Table 1. Observations and Data for some heliocentric positions for Mercury.

Date	**Radius vector (AU)**	**Longitude (°)**	**Date**	**Radius vector (AU)**	**Longitude (°)**
Oct. 1, 1990	0.319	114	Nov. 16, 1990	0.458	280
3	0.327	126	18	0.452	285
5	0.336	137	20	0.447	291
7	0.347	147	22	0.440	297
9	0.358	157	24	0.432	304
11	0.369	166	26	0.423	310
13	0.381	175	28	0.413	317
15	0.392	183	30	0.403	325
17	0.403	191	Dec.2, 1990	0.392	332
19	0.413	198	4	0.380	340
21	0.423	205	6	0.369	349
23	0.432	211	8	0.357	358
25	0.440	217	10	0.346	8
27	0.447	223	12	0.335	18
29	0.453	229	14	0.326	29
31	0.458	235	16	0.318	41
Nov. 2, 1990	0.462	241	18	0.312	53
4	0.465	246	20	0.309	65
6	0.466	251	22	0.307	78
8	0.467	257	24	0.309	90
10	0.466	262	26	0.312	102
12	0.464	268	28	0.319	114
14	0.462	273	30	0.327	126

Questions:

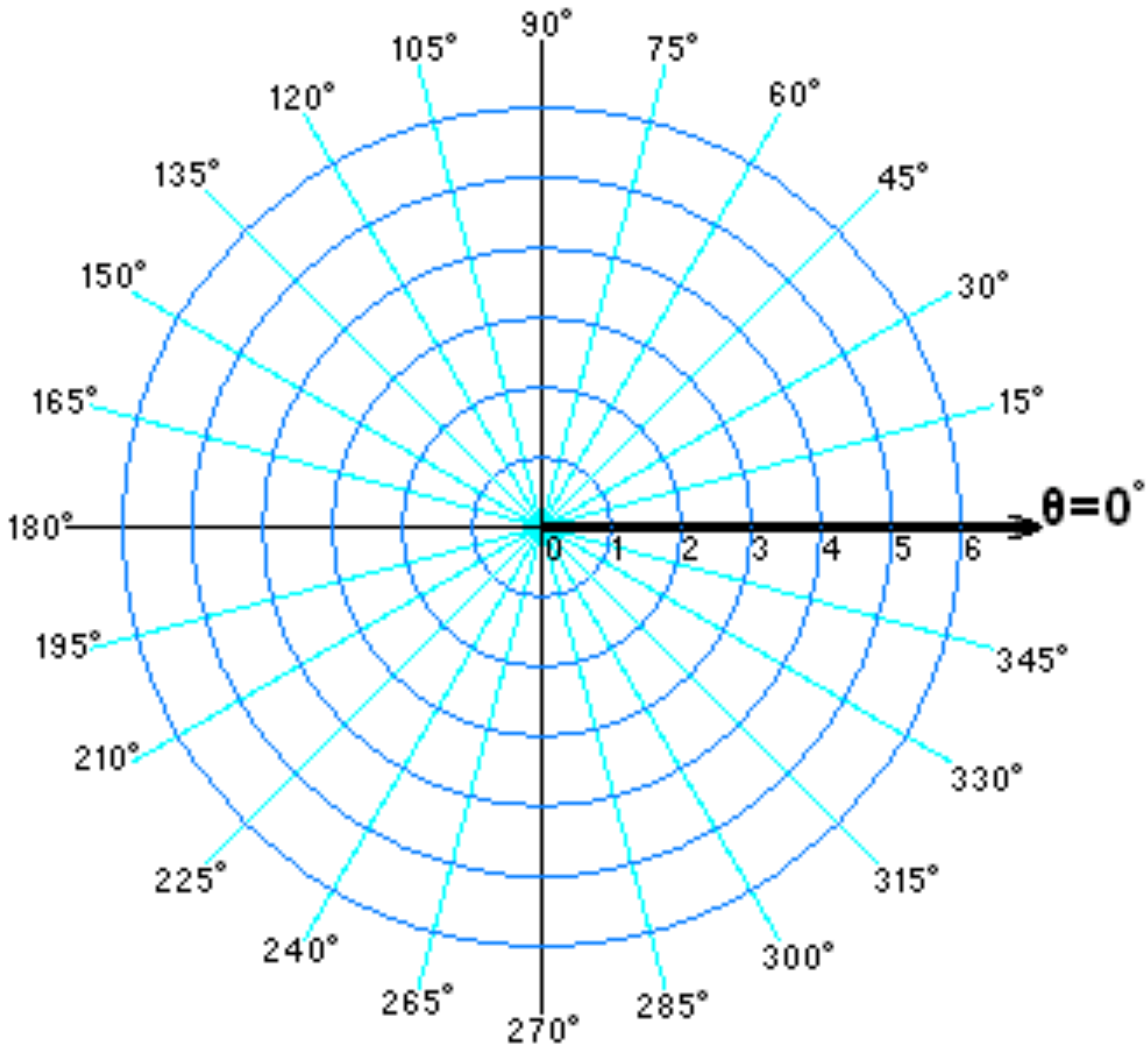
1. Does your graph of Mercury's orbit support Kepler's law of orbits?
2. Draw a line from the sun to Mercury's position on December 20. Draw a second line from the sun to Mercury's position on December 30. The two lines and Mercury's orbit describe an area swept by an imaginary line between Mercury and the sun during the ten-day interval of time. Lightly shade this area. Over a small portion of an ellipse, the area can be approximated by assuming the ellipse is similar to a circle. The equation that describes this value is: area=$(\theta/360^{o})\pi r^2$, where r is the average radius for the orbit. Determine θ by finding the difference in degrees between December 20 and 30. Measure the radius at a point midway in the orbit between the two dates. Calculate the area in AUs for this ten-day period of time.
3. Select two additional ten - day periods of time at points distant from the interval in Question 2 and shade these areas. Calculate the area in AUs for each of these ten-day periods.
4. Find the average area for the three periods of time from Questions 2 and 3. Calculate the relative error between each area and the average. Does Kepler's law of areas apply to your graph?
5. Calculate the average radius for Mercury's orbit. This can be done by averaging all the radius vectors or, more simply, by averaging the longest and shortest radii that occur along the major axis. The major axis is shown in the following figure. Recall that the sun is at one focus; the other focus is located at a point that is the same distance from the center of the ellipse as the sun, but in the opposite direction.



The major axis passes through the two foci (F and F') and the center of the ellipse. The value *ea* determines the location of the foci; *e* is the eccentricity of the orbit. If $e = 0$, the orbit is a circle, and the foci merge at one, central point.

From Table 1, find the longest radius vector. Then, align a metric ruler so that it describes a straight line passing through the point on the orbit that represents the longest radius vector and through the center of the sun to a point opposite on the orbit. Find the shortest radius vector by reading the longitude at this opposite point and consulting Table 1 for the corresponding radius vector. Average these two radius vector values. Using the values for Earth's average radius (1.0 AU), Earth period (365.25 days), and your calculated average radius of Mercury's orbit, apply Kepler's third law to find the period of Mercury. Show your calculations.

6. Refer again to the graph of Mercury's orbit that you plotted. Count the number of days required for Mercury to complete one orbit of the sun; recall that this orbital time is the period of Mercury. Is there a difference in the two values (Questions 5 and 6) for the period of Mercury? Calculate the relative difference in these two values. Are the results from you graph consistent with Kepler's law of periods?


90°
105°
75°
120°
60°
135°
45°
150°
30°
165°
15°
180°
θ=0°
0
1
2
3
4
5
6
195°
345°
210°
330°
225°
315°
240°
300°
265°
285°
270°