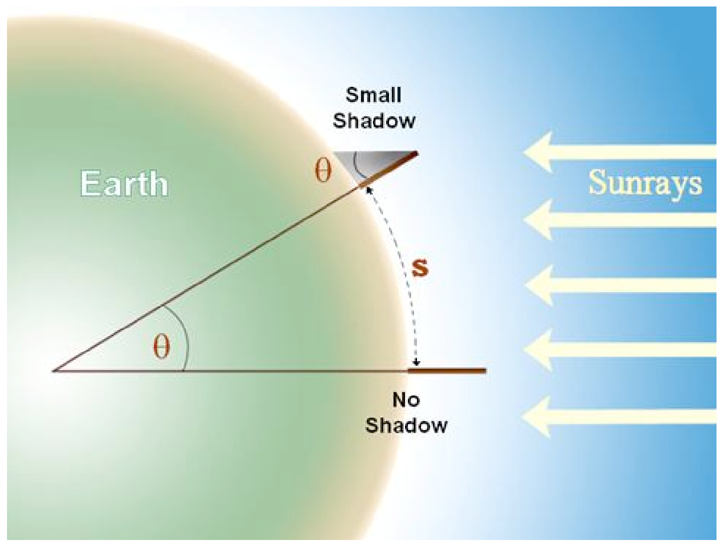
HOW BIG IS THE EARTH?
A Recreation of Erathosthenes' Experiment

PURPOSE: To measure the circumference of the Earth from the angle of the Sun on any day.

BACKGROUND: In about 300 B.C. Eratosthenes, a librarian in Alexandria, Egypt, discovered how to measure the circumference of the Earth. This is one of the most astonishing achievements of ancient science. Only about 50 years after Aristotle described the evidence that supported the idea that the Earth is shaped like a sphere, Eratosthenes figured out how to measure its circumference. In this activity, you will discover Erathsthenes' reasoning. Based on the evidence that Eratosthenes had available you will calculate the Earth's circumference.



Erathsthenes noted that at noon on the longest day of the year, the Sun's light shines directly down a well in Syene, a city that is several hundred miles to the south of Alexandria. When he looked at a vertical post in Alexandria at noon on the longest day of the year, the Sun's rays cast a shadow 1/8 the length of the post.

From Euclid's geometry, he found that the angle at the top of the post must have been about 1/50 of a circle (a little more than 7 degrees). When he drew a diagram of the Earth (see figure) he realized that the angle of the shadow cast by the vertical post in Alexandria equals an imaginary angle formed by the center of the Earth, Alexandria and Syene, since the angle was 1/50 of a circle, the distance between Alexandria and Syene must be 1/50 of the distance around the Earth.

He then paid someone to measure the distance from Syene to Alexandria by walking from one city to the other, and counting his steps. He measured the distance to be 5,000 stadia (about 493 miles in modern terms).

To calculate the circumferenc of the Earth we don't have to wait until the longest day of the year (June 21). Every day there is a spot on the Earth where the sun is directly overhead at noon. We just need to know where on the Earth this occurs. The table found in the next few pages will give you this information. It is a list of dates and Declinations for the Sun. Next look on a map and find the latitude of your city. We are at 33.5^0 N latitude. Now all you need to know is the distance between latitudes. Use a map to obtain this information.

WORKSHEET: CALCULATION OF THE CIRCUMFERENCE:

DATA:
1) Angle of the Sun ________________________________degrees

2) Distance from one latitude to another ________________________in miles

3) Latitude where the Sun is directly overhead _________________________degrees

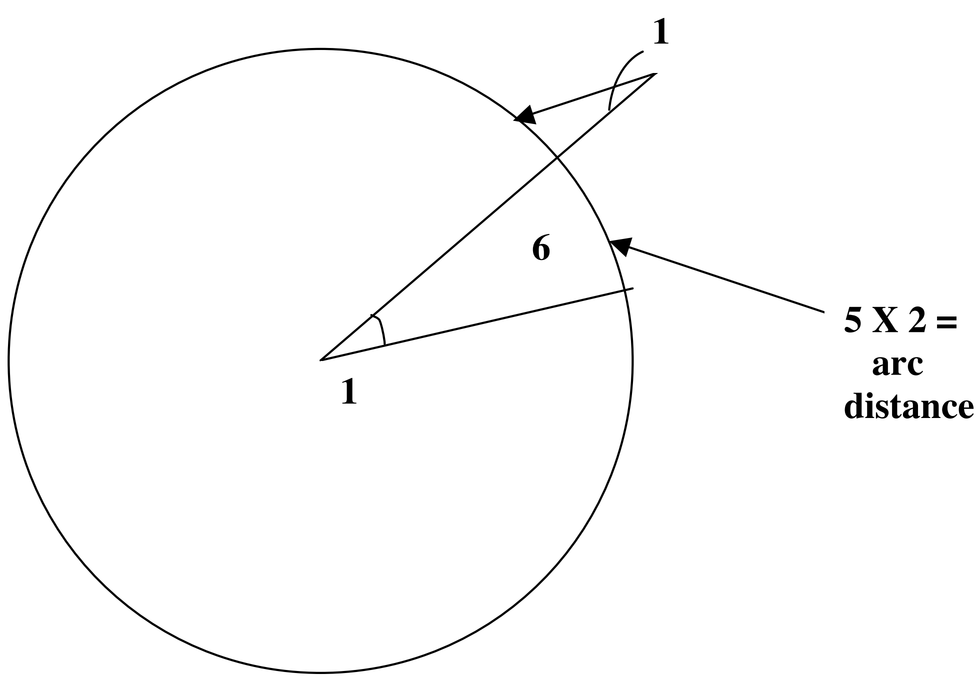
4) My latitude __________________________________degrees

5) Difference between the latitudes _____________________________degrees

6) $\frac{360}{\text{angle of the sun (1)}}$ = _________________ (pieces of the "pie" to make a whole)

7) Answer from (6) x (5) x (2) = ______________________________ miles

8) Find your % error ____



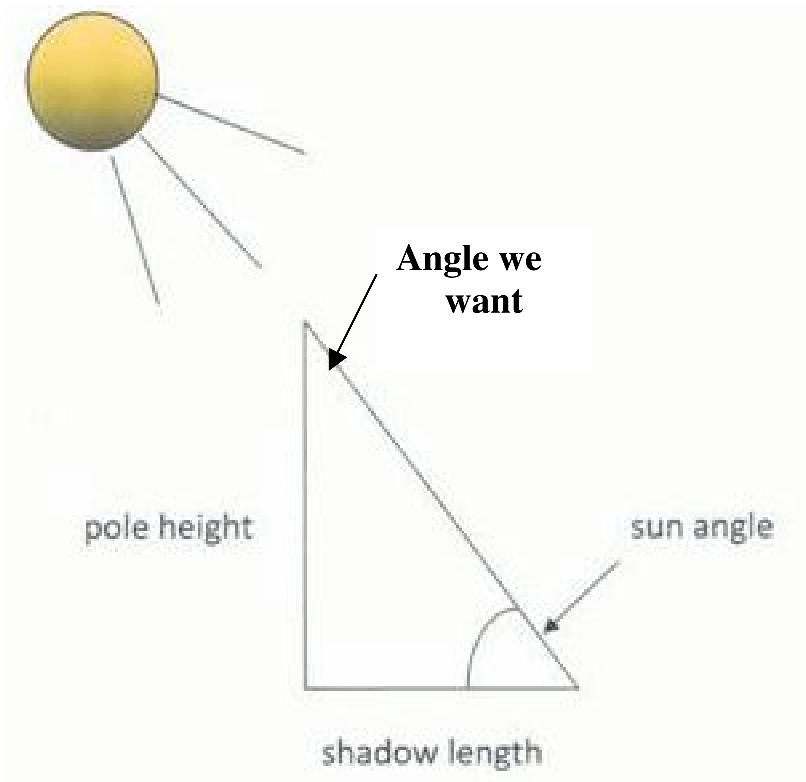
APPENDICES FOR THOSE DOING THE LAB AT HOME

APPENDIX 1 HOW TO FIND THE ANGLE OF THE SUN

You will need to set up a rod (or similar straight object). This should be positioned so that it is VERY straight up and down and the shadow points in the northerly direction. You are going to plot the shadow for the two hours around HIGH NOON (12ish if we are on Standard time and 1ish if we are on Day Light Savings Time). You will need a large sheet of paper or several taped together so that the shadow falls on the paper. Record the position of the top of the shadow at 10 minute intervals until you get to HIGH NOON and then two minute intervals. Also record the time at each mark.

Your shadow will shorten then lengthen. The shortest point for the shadow is the time when the sun was crossing your meridian. This is the highest the sun will be on that particular day. To find the angle of the sun measure the height of the pole and the length of the shadow in any unit you want (cm or inches). Use the following equation to find the angle the sun makes with the top of the pole.

$$\text{angle} = \arctan\ (\text{shadow} / \text{pole})$$



APPENDIX 2 HOW TO FIND THE DISTANCE BETWEEN LATTITUDES.

Using the Google maps find 33 N 117 W and 34 N 117 W. Make the two locations appear on the same map. Print the map so that the two locations are shown fairly far apart and the scale at the bottom of the map is printed. If the scale won't print on the map you may have to do a screen capture and print that. Using the scale find the distance between the two points. You have found the distance between two latitude lines. This is the easiest measurement to mess up and created a large % error so be careful.

APPENDIX 3 IS FOUND ONLY IN THE PDF VERSION AND IS A TABLE OF SUN DECLINATIONS ON PARTICULAR DAYS.

Table of the Declination of the Sun

Mean Value for the Four Years of a Leap-Year Cycle

Positive sign (+) Sun north of Celestial Equator; negative sign (-) Sun south of Celestial Equator.

Day	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1	-23°04'	-17°20'	-7°49'	+4°18'	+14°54'	+21°58'	+23°09'	+18°10'	+8°30'	-2°57'	-14°14'	-21°43'
2	-22°59'	-17°03'	-7°26'	+4°42'	+15°12'	+22°06'	+23°05'	+17°55'	+8°09'	-3°20'	-14°34'	-21°52'
3	-22°54'	-16°46'	-7°03'	+5°05'	+15°30'	+22°14'	+23°01'	+17°40'	+7°47'	-3°44'	-14°53'	-22°01'
4	-22°48'	-16°28'	-6°40'	+5°28'	+15°47'	+22°22'	+22°56'	+17°24'	+7°25'	-4°07'	-15°11'	-22°10'
5	-22°42'	-16°10'	-6°17'	+5°51'	+16°05'	+22°29'	+22°51'	+17°08'	+7°03'	-4°30'	-15°30'	-22°18'
6	-22°36'	-15°52'	-5°54'	+6°13'	+16°22'	+22°35'	+22°45'	+16°52'	+6°40'	-4°53'	-15°48'	-22°25'
7	-22°28'	-15°34'	-5°30'	+6°36'	+16°39'	+22°42'	+22°39'	+16°36'	+6°18'	-5°16'	-16°06'	-22°32'
8	-22°21'	-15°15'	-5°07'	+6°59'	+16°55'	+22°47'	+22°33'	+16°19'	+5°56'	-5°39'	-16°24'	-22°39'
9	-22°13'	-14°56'	-4°44'	+7°21'	+17°12'	+22°53'	+22°26'	+16°02'	+5°33'	-6°02'	-16°41'	-22°46'
10	-22°05'	-14°37'	-4°20'	+7°43'	+17°27'	+22°58'	+22°19'	+15°45'	+5°10'	-6°25'	-16°58'	-22°52'
Day	**JAN**	**FEB**	**MAR**	**APR**	**MAY**	**JUN**	**JUL**	**AUG**	**SEP**	**OCT**	**NOV**	**DEC**
11	-21°56'	-14°18'	-3°57'	+8°07'	+17°43'	+23°02'	+22°11'	+15°27'	+4°48'	-6°48'	-17°15'	-22°57'
12	-21°47'	-13°58'	-3°33'	+8°28'	+17°59'	+23°07'	+22°04'	+15°10'	+4°25'	-7°10'	-17°32'	-23°02'
13	-21°37'	-13°38'	-3°10'	+8°50'	+18°14'	+23°11'	+21°55'	+14°52'	+4°02'	-7°32'	-17°48'	-23°07'
14	-21°27'	-13°18'	-2°46'	+9°11'	+18°29'	+23°14'	+21°46'	+14°33'	+3°39'	-7°55'	-18°04'	-23°11'
15	-21°16'	-12°58'	-2°22'	+9°33'	+18°43'	+23°17'	+21°37'	+14°15'	+3°16'	-8°18'	-18°20'	-23°14'
16	-21°06'	-12°37'	-1°59'	+9°54'	+18°58'	+23°20'	+21°28'	+13°56'	+2°53'	-8°40'	-18°35'	-23°17'
17	-20°54'	-12°16'	-1°35'	+10°16'	+19°11'	+23°22'	+21°18'	+13°37'	+2°30'	-9°02'	-18°50'	-23°20'
18	-20°42'	-11°55'	-1°11'	+10°37'	+19°25'	+23°24'	+21°08'	+13°18'	+2°06'	-9°24'	-19°05'	-23°22'
19	-20°30'	-11°34'	-0°48'	+10°58'	+19°38'	+23°25'	+20°58'	+12°59'	+1°43'	-9°45'	-19°19'	-23°24'
20	-20°18'	-11°13'	-0°24'	+11°19'	+19°51'	**+23°26'**	+20°47'	+12°39'	+1°20'	-10°07'	-19°33'	-23°25'
Day	**JAN**	**FEB**	**MAR**	**APR**	**MAY**	**JUN**	**JUL**	**AUG**	**SEP**	**OCT**	**NOV**	**DEC**
21	-20°05'	-10°52'	**0°00'**	+11°39'	+20°04'	**+23°26'**	+20°36'	+12°19'	+0°57'	-10°29'	-19°47'	**-23°26'**
22	-19°52'	-10°30'	+0°24'	+12°00'	+20°16'	**+23°26'**	+20°24'	+11°59'	+0°33'	-10°50'	-20°00'	**-23°26'**
23	-19°38'	-10°08'	+0°47'	+12°20'	+20°28'	**+23°26'**	+20°12'	+11°39'	**+0°10'**	-11°12'	-20°13'	**-23°26'**
24	-19°24'	-9°46'	+1°11'	+12°40'	+20°39'	+23°25'	+20°00'	+11°19'	-0°14'	-11°33'	-20°26'	**-23°26'**
25	-19°10'	-9°24'	+1°35'	+13°00'	+20°50'	+23°24'	+19°47'	+10°58'	-0°37'	-11°54'	-20°38'	-23°25'
26	-18°55'	-9°02'	+1°58'	+13°19'	+21°01'	+23°23'	+19°34'	+10°38'	-1°00'	-12°14'	-20°50'	-23°23'
27	-18°40'	-8°39'	+2°22'	+13°38'	+21°12'	+23°21'	+19°21'	+10°17'	-1°24'	-12°35'	-21°01'	-23°21'
28	-18°25'	-8°17'	+2°45'	+13°58'	+21°22'	+23°19'	+19°08'	+9°56'	-1°47'	-12°55'	-21°12'	-23°19'
29	-18°09'	-8°03'	+3°09'	+14°16'	+21°31'	+23°16'	+18°54'	+9°35'	-2°10'	-13°15'	-21°23'	-23°16'
30	-17°53'		+3°32'	+14°35'	+21°41'	+23°13'	+18°40'	+9°13'	-2°34'	-13°35'	-21°33'	-23°12'
31	-17°37'		+3°55'		+21°50'		+18°25'	+8°52'		-13°55'		-23°08'