Disclaimer

You should use this practice exam to assess your speed and to improve your ability to correctly identify different problem types. The questions on this practice exam are taken from exams given in previous semesters, but they may not be representative of the questions that will appear on this semester's exam. You should also invest time re-reading the relevant parts of your textbook, reviewing your notes, and practicing homework problems.
You will not receive full credit if you do not clearly show how you are obtaining your answers. Show all work on this exam; do not attach other work. Circle your answers.

1. Find the horizontal or oblique asymptote of \( H(x) = \frac{5x^4 + 7x - 3}{x^3 + x^2 - 2x + 1} \). (6 points)

Write *none* if neither asymptote exists.

\[
\begin{align*}
5x - 5 \\
\overbrace{x^3 + x^2 - 2x + 1}^{5x^4 + 0x^3 + 0x^2 + 7x - 3} \\
\overbrace{5x^4 + 5x^3 - 10x^2 + 5x} \\
-5x^3 + 10x^2 + 2x - 3 \\
-5x^3 - 5x^2 + 10x - 5 \\
15x^2 - 8x - 8
\end{align*}
\]

So the oblique asymptote is \( y = 5x - 5 \).

2. Solve and graph the solution set: \( \frac{(x + 5)}{(x - 4)(x + 2)^2} \leq 0 \). (14 points)

Write your answer in interval notation.

<table>
<thead>
<tr>
<th>Test</th>
<th>( \frac{(x + 5)}{(x - 4)(x + 2)^2} )</th>
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</tr>
</thead>
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<td>False</td>
<td>False</td>
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<tr>
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<td>5</td>
<td>False</td>
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</tbody>
</table>

So the solution set is \((-5, -2) \cup (-2, 4)\)
Consider the function \( f(x) = x^2 + 4x + 3 \). (20 points)

a. Find the coordinates of the vertex. Plot the vertex on the coordinate plane provided.

\[
h = -\frac{b}{2a} = -\frac{4}{2(1)} = -2 \\
 k = f\left(-\frac{b}{2a}\right) = f(-2) = (-2)^2 + 4(-2) + 3 = -1
\]

So the vertex has the coordinates (-2, -1).

b. Find the equation of the axis of symmetry. Graph the axis on the coordinate plane.

\[x = -2\]

c. Find all \( x \) and \( y \) intercepts.

\[y\text{-int}: f(0) = (0)^2 + 4(0) + 3 = 3.\] So the \( y \)-int. is the point (0, 3).

\[x\text{-int}
\]

\[
0 = x^2 + 4x + 3 \\
0 = (x + 3)(x + 1)
\]

So the \( x \)-intercepts are (-3, 0) and (-1, 0).

d. State the domain and range of the function in interval notation.

\[
D : (-\infty, \infty) \\
R : [-1, \infty)
\]

e. Graph the function. *Plot at least three points in addition to the vertex and intercepts.*
4. Let \( f(x) = \frac{x^2 - 5x + 4}{x^2 - 2x - 8} = \frac{(x - 4)(x - 1)}{(x - 4)(x + 2)} = \frac{x - 1}{x + 2} \), where \( x \neq 4 \). (19 points)

a. Write the equations for any vertical asymptotes. If there aren’t any then write *none*.

The denominator is zero if \( x + 2 = 0 \), so there is a vertical asymptote at \( x = -2 \)

b. Write the equation for the horizontal or oblique asymptote. If there isn’t one then write *none*.

The numerator and denominator have the same degree so there is a horizontal asymptote at the ratio of the leading coefficients \( y = 1 \).

c. Find the coordinates of any “holes” in the function. If there aren’t any then write *none*.

\[
\begin{align*}
x &= 4 \\
y &= \frac{x - 1}{x + 2} = \frac{4 - 1}{4 + 2} = \frac{3}{6} = \frac{1}{2}
\end{align*}
\]

So there is a hole at \( \left( 4, \frac{1}{2} \right) \).

d. Find all intercepts for the graph of \( f(x) \).

y-int: \( f(0) = \frac{0 - 1}{0 + 2} = -\frac{1}{2} \)
So the y-int is \( \left( 0, -\frac{1}{2} \right) \).

x-int: The numerator is zero when \( x - 1 = 0 \) so the x-int. is \( (1, 0) \).

e. Accurately graph \( f(x) \). Be certain to draw all asymptotes, intercepts, and “holes.” *Plot additional points on the graph if necessary.*
Consider the polynomial function \( f(x) = -(0.1)(x - 5)(x + 2)^2 \). (17 points)

a. Find any \( x \)-intercepts of \( f(x) \), and determine whether the graph of \( f \) crosses or touches the \( x \)-axis at each \( x \)-intercept.

There is an \( x \)-int. at \((5, 0)\) with a multiplicity of 1, so the graph crosses at \( x = 5 \).

There is an \( x \)-int. at \((-2, 0)\) with a multiplicity of 2, so the graph touches at \( x = -2 \).

b. Determine the behavior of the graph near \( x = 5 \).

Near \( x = 5 \), \( f(x) \approx -(0.1)(x - 5)(5 + 2)^2 = -(0.1)(49)(x - 5) = -4.9x + 24.5 \)

c. End behavior: find the power function that the graph of \( f \) resembles for large values of \( |x| \).

\[ f(x) = -(0.1)(x - 5)(x + 2)^2 = -(0.1)x^3 + \ldots \] so the power function is \(-0.1x^3\).

d. Determine the maximum number of turning points on the graph of \( f(x) \).

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e. Find the \( y \)-intercept of \( f(x) \).

\[ f(0) = -(0.1)(0 - 5)(0 + 2)^2 = (0.1)20 = 2 \]

So \( y \)-int: \((0, 2)\).

f. Sketch the graph of \( f(x) \) on the coordinate plane. If necessary, find a few additional points on the graph.
6. Form a third degree polynomial \( f(x) \) with real coefficients and -2, -1 and 3 as zeros. Write the polynomial in standard form: \( f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0. \) (10 points)

\[
f(x) = [x - (-2)][x - (-1)](x - 3) \\
= (x + 2)(x + 1)(x - 3) \\
= (x^2 + 3x + 2)(x - 3) \\
= x^3 + 3x^2 + 2x - 3x^2 - 9x - 6 \\
= x^3 - 7x - 6
\]

7. The price \( p \) (in dollars) and the quantity \( x \) sold of a certain product obey the demand equation \( p = -\frac{1}{3}x + 600, 0 \leq x \leq 1800. \) (14 points)

a. Express the revenue \( R \) as a function of \( x. \)

\[
R(x) = xp = x\left(-\frac{1}{3}x + 600\right) = -\frac{1}{3}x^2 + 600x
\]

b. What quantity should be sold to maximize the revenue?

\[
-\frac{b}{2a} = -\frac{600}{2(-1/3)} = \frac{600}{2(2/3)} = 600\left(\frac{3}{2}\right) = 900
\]

c. What is the maximum revenue for this product?

\[
R\left(-\frac{b}{2a}\right) = R(900) = -\frac{1}{3}(900)^2 + 600(900) = $270,000
\]

d. What price should be charged to achieve the maximum revenue?

\[
p = -\frac{1}{3}(900) + 600 = -300 + 600 = $300
\]