Vectors

Saddleback College Physics Department

Purpose

To analytically and experimentally add and subtract vectors, verifying that these sums correspond.

Theory

What is a Vector?

A vector is a physical parameter that has a direction and magnitude. A classic vector quantity is velocity. The magnitude of the velocity is the speed. As an example, a car may be traveling 50 mph (speed) due north (direction). Vectors are represented by bold letters (\( \mathbf{v} \)) or a letter with an arrow over the top (\( \vec{v} \)). Unlike vectors, scalar quantities only have a magnitude.

Vector components

Vectors can be resolved into components. The number of components is proportional to the number of spacial dimensions. If operating in two dimensions (x, y coordinates), the vector will have two components: an x and a y component (see Figure 2). As an example, a car traveling north-east has a northerly component and an easterly component. The vector components are always parallel to the axis (see Figure 1). As shown in Figure 1, simple trigonometry can be used to resolve vectors into components. Please note that the angular direction of the vector is specified with respect to the x axis.

Magnitude of \( \mathbf{A} \):

\[
\mathbf{A} = \sqrt{A_x^2 + A_y^2}
\]

\( A_x = A \cos \theta \)

\( A_y = A \sin \theta \)

The magnitude and direction of the vector \( \mathbf{A} \) is shown in Figure 1.

Figure 1. A vector has a magnitude and a direction.

Figure 2. Vectors can be resolved into components. The vector \( \mathbf{A} \) is resolved into \( x \) \( (A_x) \) and \( y \) \( (A_y) \) components.

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Vector Addition

In some instances we may want to add two vectors to calculate the net magnitude and direction traveled. Two methods are available for adding vectors: analytical methods (such as the Law of Cosines or Component method) or graphical methods (such as the Parallelogram or Tip-To Tail method) several of which are described below.

**GRAPHICAL METHOD OF VECTOR ADDITION:**

Graphically adding vectors requires a ruler, protractor and possibly graph paper. Each vector is scaled appropriately and added by aligning the vectors “head to tail” (see Figure 3). The net vector is called the resultant. This graphical method is effective and simple if care is taken in scaling and drawing the vectors. Furthermore, the graphical method increases in complexity when more than 2 vectors are added.

\[ A + B = C \] (Resultant)

![Figure 3. Graphical method of adding vectors](image)

**COMPONENT METHOD OF VECTOR ADDITION:**

The component method is a “cookbook” process that can be used to add any number of vectors. It simply involves adding all of the like components and using the Pythagorean theorem. Here is the process:

1. Place the two vectors on a Cartesian coordinate system. Angles \( \theta \) and \( \phi \) are measured with respect to the x-axis.

   ![Diagram](image)

2. Resolve each vector into components

   \[ A_x = A \cos \phi \quad A_y = A \sin \phi \quad B_x = B \cos \theta \quad B_y = B \sin \theta \]

3. Add all like components

   \[ C_x = A_x + B_x \quad C_y = A_y + B_y \]

4. Use the Pythagorean theorem to calculate the magnitude

   \[ C = \sqrt{C_x^2 + C_y^2} \]

5. Calculate the final direction

   \[ \text{angle} = \tan^{-1}\left( \frac{C_y}{C_x} \right) \]; this angle is with respect to the x-axis ONLY. In this case C is between A and B.

6. Subtracting vectors is performed in the same manner except one must reverse the direction of the negative vector.
Procedure

First you will add two vectors analytically and later you will add them experimentally using a force table.

1. For each of the three trials in the chart below, you will add vectors $\vec{A}$ & $\vec{B}$ analytically using the component method, thus obtaining the magnitude and direction of the resultant vector $\vec{C}$. Be sure you show complete calculations for all three trials in the calculations section of your lab write-up.

[NOTE: In this lab we will add force vectors since this quantity is easily obtained using simple mechanisms. We will formally introduce force at a later date. For now, just treat the force as a vector quantity: $F=mg$, where $m$ is mass and $g$ is the acceleration of gravity in Mission Viejo ($g = 9.80 \text{ m/s}^2$). The SI unit for Force is "Newton" and 1 Newton = $\frac{1 \text{ kg} \cdot \text{m}}{\text{s}^2}$.]

2. Use the force table to verify your calculations. The force table (Figure 4.) consists of a large graduated disc, string, weights and a balancing ring. A vector is simulated by hanging a weight (corresponding to the magnitude of the vector) from the string at the appropriate angle (corresponding to the direction of the vector).

   a) Use a pencil to mark the x, y coordinate axes on the disc. The +x coordinate axis should be at 0 degrees.
   b) Place the two weights at the appropriate angles.
   c) Now balance the weights using a third weight placed on the opposite side of the force table. You will have to use trial and error to adjust this third weight (representing vector "C") so that it balances the system. The system is balanced when the supporting ring at the center of the force table is centered on the peg.
   d) Once the ring is centered, record the third weight and its angle, CALLING THIS VECTOR "-C" since it is equal in magnitude to the vector sum "C" but opposite in direction. Notice that all vectors are shown in red in Figure 4.
   e) Compile all of your results (magnitudes and directions) in a table.

![Figure 4. The force table is used to experimentally add two vectors](image-url)
**Analysis:** Calculate the percent difference between your analytical and experimental results for both **magnitude** and **direction**. Are they significantly different?

Make sure to record any procedural errors, which may explain the difference in your results. Your results might be summarized nicely (in your conclusion) in a table such as the one below.

<table>
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<th>Trial #</th>
<th>Analytical (theoretical)</th>
<th>Experimental</th>
<th>% difference in C magnitude</th>
<th>% difference in C angle</th>
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<tr>
<td></td>
<td>C magnitude (N)</td>
<td>C magnitude (N)</td>
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<td></td>
<td>C angle (degrees)</td>
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