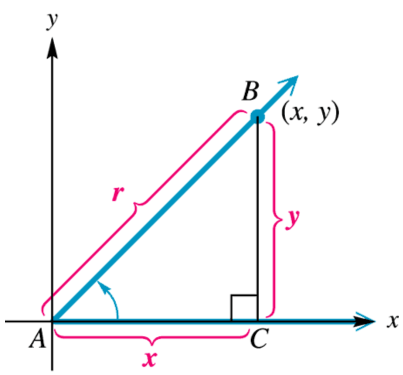
Section 2.1: Trigonometric Functions of Acute Angles

I. Right Triangle Based Definitions

The six major trigonometric functions of an angle θ , sine, cosine, tangent, cosecant, secant, and tangent, can easily be **redefined** as *ratios of the lengths of the sides of right triangles*. We can do this by talking about the **side opposite** an angle, the **side adjacent** to an angle, and the **hypotenuse**. Let's explore this now!



Right-Triangle-Based Definitions of Trigonometric Functions

Let A represent any acute angle in standard position.

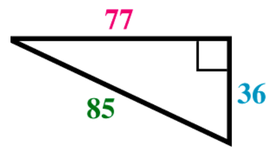
$$\sin A = \frac{y}{r} = \frac{\text{side opposite } A}{\text{hypotenuse}} \qquad \csc A = \frac{r}{y} = \frac{\text{hypotenuse}}{\text{side opposite } A}$$

$$\cos A = \frac{x}{r} = \frac{\text{side adjacent to } A}{\text{hypotenuse}} \qquad \sec A = \frac{r}{x} = \frac{\text{hypotenuse}}{\text{side adjacent to } A}$$

$$\tan A = \frac{y}{x} = \frac{\text{side opposite } A}{\text{side adjacent to } A} \qquad \cot A = \frac{x}{y} = \frac{\text{side adjacent to } A}{\text{side opposite } A}$$

- The mnemonic **Soh/Cah/Toa**

Example 1 (Finding Function Values): Find the sine, cosine, and tangent values for angles *A* and *B* in the figure.



II. Cofunctions

Notice in the previous example there was a connection between the sine of angle *A* and the cosine of angle *B*, the cosine of angle *A* and the sine of angle *B*, etc….

- These connections are known as the **cofunction identities**. Let's explore these in general.

Cofunction Identities

For any acute angle A, cofunction values of complementary angles are equal.

$$\sin A = \cos(90° - A) \qquad \cos A = \sin(90° - A)$$

$$\tan A = \cot(90° - A) \qquad \cot A = \tan(90° - A)$$

$$\sec A = \csc(90° - A) \qquad \csc A = \sec(90° - A)$$

.

Example 2 (Rewriting Functions in Terms of Cofunctions): Write each function in terms of its cofunction.

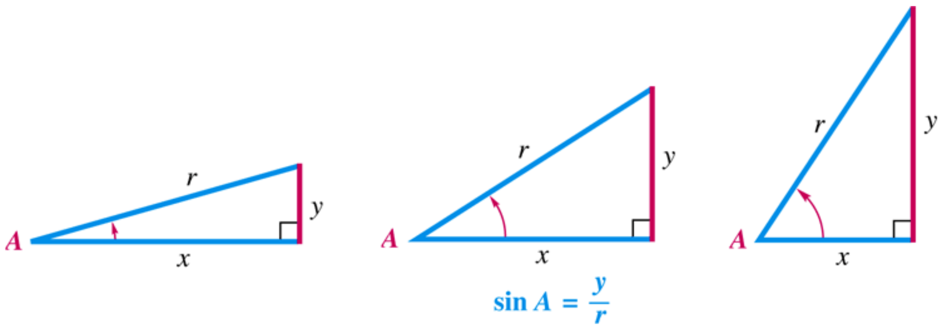
(a) sin 9°

(b) cot 76°

(c) csc 45°

III. Increasing and Decreasing Functions

- Note that as an angle *A* increases from 0° to 90°, *y* increases, *x* decreases, and r is always fixed.



Example 3 (Comparing Function Values): Decide whether each statement is *true* or *false.*

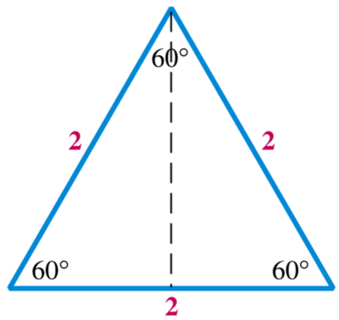
(a) $\sin 21° > \sin 18°$

(b) $\tan 25° < \tan 23°$

(c) $\csc 44° < \csc 40°$

IV. Special Angles

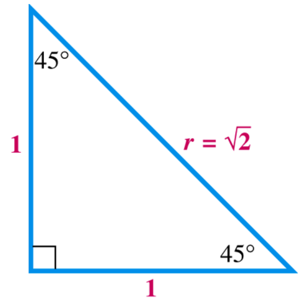
- The **30°-60°-90°** triangles. To start, let’s bisect one angle of an equilateral triangle.



Equilateral triangle

Example 4 (Finding values for 30°): Find the six trigonometric function values for a 30° angle.

- **45°- 45°-90°** Right Triangles.



45°– 45° right triangle

Example 5 (Practice): Now you find the six values for 45° angle.

Function Values of Special Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°			
60°			
45°			