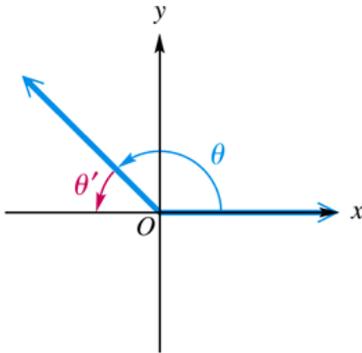


Section 2.2: Trigonometric Functions of Non-Acute Angles

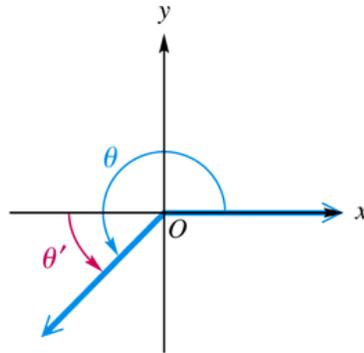
I. Reference Angles

For any nonquadrantal angle in standard position there exists a very important positive acute angle known as a *reference angle*.

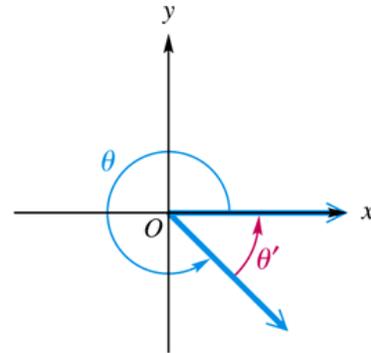
- A **reference angle** for an angle θ is the positive acute angle made by the terminal side of angle θ and the x -axis.
- Here are the possibilities for quadrants II, III, IV. (Why not quadrant I?)



θ in quadrant II



θ in quadrant III



θ in quadrant IV

Caution

A common error is to find the reference angle by using the terminal side of θ and the y -axis.

The reference angle is always found with reference to the x -axis.

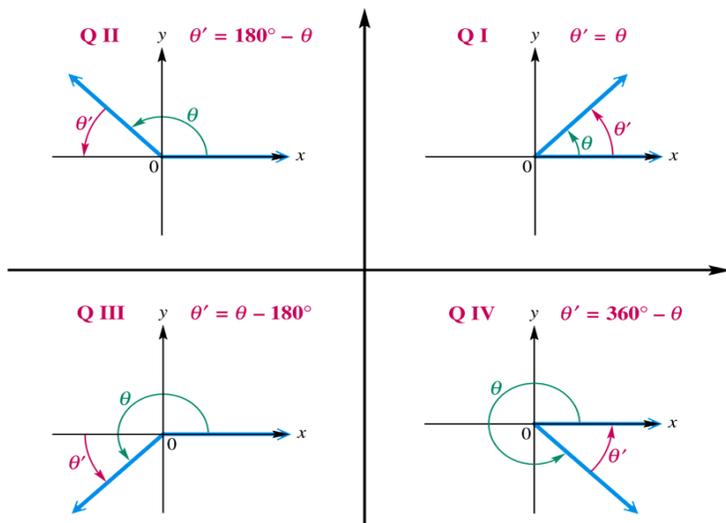
Example 1 (Finding Reference Angles): Find the reference angles for the following:

a) 294°

b) 883°

The previous examples suggest a pattern in finding reference angles based on their quadrant.

Reference Angle θ' for θ , where $0^\circ < \theta < 360^\circ$

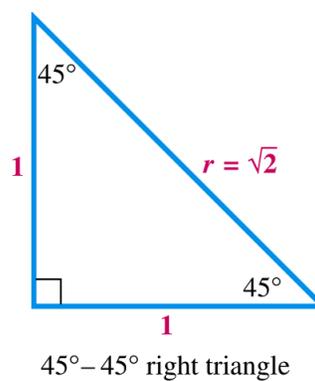
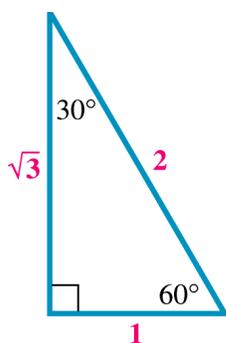


Side Notes:

- Always find a coterminal angle between 0° and 360° .

II. Special Angles as Reference Angles

- We can now find the **exact value of trigonometric functions** with reference angles of 30° , 45° , and 60° . This is a BIG DEAL!!!
- Here is a reminder of our two *Special Triangles* and their angles.



Finding Trigonometric Function Values For Any Nonquadrantal Angle θ

- STEP 1:** If $\theta > 360^\circ$, or if $\theta < 0^\circ$, find a coterminal angle by adding or subtracting 360° as many times as needed to get an angle greater than 0° but less than 360° .
- STEP 2:** Find the reference angle θ' .
- STEP 3:** Find the trigonometric function values for reference angle θ' .
- STEP 4:** Determine the correct signs for the values found in *Step 3*. This gives the values of the trigonometric functions for angle θ .

Example 2 (Finding Trigonometric Functions of a Quadrant III Angle): Find the exact values of the $\sin 225^\circ$, $\cos 225^\circ$, and $\tan 225^\circ$.

Example 3 (Finding Trigonometric Functions Values Using Reference Angles): Find the exact value of $\tan 675^\circ$.

Practice: Find the exact value of $\cos(-240^\circ)$.

III. Finding Angle Measures with Special Angles

We can use the ideas of *reference angles* and *special values* to start solving some simple trigonometric equations. Here are some tips:

- Pay close attention to the *sign* of the trigonometric value and thus its quadrant(s). (ASTC)
- Always associate *cosine* to values on the x -axis.
- Always associate *sine* to values on the y -axis.

Example 4 (Solving Equations): Find all values of θ , if θ is in the interval $[0^\circ, 360^\circ)$ and

$$\sin \theta = -\frac{\sqrt{3}}{2}.$$