

Section 4.3: The Graphs of Tangent and Cotangent

I. Vertical Asymptotes

- **Definition:** A **vertical asymptote** is a vertical line that the graph approaches but does not intersect. As the x -values get closer and closer to the line, the function values increase or decrease *without bound*.

Example: Let's examine the graph of $y = 1/x$ near $x = 0$.

x	y
0	

II. The Tangent Function

x	y
0	

Facts about the graph of $f(x) = \tan x$.

- The graph is **discontinuous** at values of x of the form $x = \frac{\pi}{2} + n\pi, n = 0, \pm 1, \pm 2, \dots$, and has **vertical asymptotes** at these values.
- Its **period** is $p = \pi$.
- Its graph has **no amplitude**, since there are no minimum or maximum values.
- The graph is **symmetric with respect to the origin**, so the function is an odd function. For all x in the domain, $\tan(-x) = -\tan(x)$
- **Domain:**
- **Range:**

III. The Cotangent Function

x	y
0	

Facts about the graph of $f(x) = \cot x$.

- The graph is **discontinuous** at values of x of the form $x = 0 + n\pi, n = 0, \pm 1, \pm 2, \dots$, and has **vertical asymptotes** at these values.
- Its **period** is $p = \pi$.
- Its graph has **no amplitude**, since there are no minimum or maximum values.
- The graph is **symmetric with respect to the origin**, so the function is an odd function. For all x in the domain, $\cot(-x) = -\cot(x)$.
- **Domain:**
- **Range:**

IV. Graphing Techniques

Guidelines for Sketching Graphs of Tangent and Cotangent Functions

- STEP 1:** Determine the period, $\frac{\pi}{b}$. To locate two adjacent vertical asymptotes, solve the following equations for x :
- For $y = a \tan bx$: $bx = -\frac{\pi}{2}$ and $bx = \frac{\pi}{2}$
 For $y = a \cot bx$: $bx = 0$ and $bx = \pi$
- STEP 2:** Sketch the two vertical asymptotes found in Step 1.
- STEP 3:** Divide the interval formed by the vertical asymptotes into four equal parts.
- STEP 4:** Evaluate the function for the first-quarter point, midpoint, and third-quarter point, using the x -values found in Step 3.
- STEP 5:** Join the points with a smooth curve, approaching the vertical asymptotes. Indicate additional asymptotes and periods of the graph as necessary.

Example 1: Graph $y = \tan \frac{2}{3}x$.

Example 2: Graph $y = -\frac{1}{2}\tan(2x)$

Example 3: Graph $y = 3 \cot\left(\frac{1}{2}x\right)$

Example 4 (Graphing a Tangent Function With a Vertical Translation):

Graph $y = -3 + \tan x$.