

Section 4.3: The Graphs of Secant and Cosecant

I. The Secant Function

- The easiest way to graph a secant function is to think of it as the reciprocal of cosine.

$$\sec x = \frac{1}{\cos x}$$

x	y

Facts about the graph of $f(x) = \sec x$.

- The graph is **discontinuous** at values of x of the form $x = \frac{\pi}{2} + n\pi, n = 0, \pm 1, \pm 2, \dots$, and has **vertical asymptotes** at these values.
- There are **no x-intercepts**.
- Its **period** is $p = 2\pi$.
- Its graph has **no amplitude**, since there are no minimum or maximum values.
- The graph is **symmetric with respect to the y-axis**, so the function is an even function. For all x in the domain, $\sec(-x) = \sec(x)$
- Domain:**
- Range:**

II. The Cosecant Function

- The easiest way to graph a cosecant function is to think of it as the reciprocal of sine.

$$\csc x = \frac{1}{\sin x}$$

x	y

Facts about the graph of $f(x) = \csc x$.

- The graph is **discontinuous** at values of x of the form $x = 0 + n\pi, n = 0, \pm 1, \pm 2, \dots$, and has **vertical asymptotes** at these values.
- There are **no x-intercepts**.
- Its **period** is $p = 2\pi$.
- Its graph has **no amplitude**, since there are no minimum or maximum values.
- The graph is **symmetric with respect to the origin**, so the function is an odd function. For all x in the domain, $\csc(-x) = -\csc(x)$
- Domain:**
- Range:**

III. Graphing Techniques**Guidelines for Sketching Graphs of
Cosecant and Secant Functions**

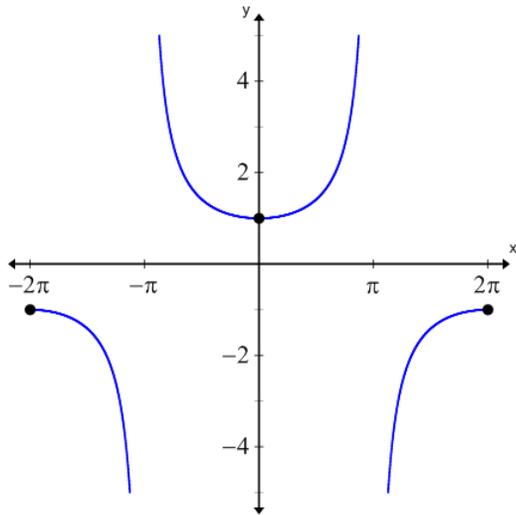
- STEP 1:** Graph the corresponding reciprocal function as a guide, using a dashed curve.
- STEP 2:** Sketch the vertical asymptotes. They will have equations of the form $x = k$, where k is an x -intercept of the graph of the guide function.
- STEP 3:** Sketch the graph of the desired function by drawing the typical U-shaped branches between the adjacent asymptotes.
- STEP 4:** The branches will be above the graph of the guide function when the guide function values are positive and below the graph of the guide function when the guide function values are negative.

Example 1: Graph $y = 3\sec 2x$.

Example 2: Graph $y = \frac{1}{2} \csc\left(x + \frac{\pi}{4}\right)$

Example 3: Determine an equation for the following graphs.

a)



b)

