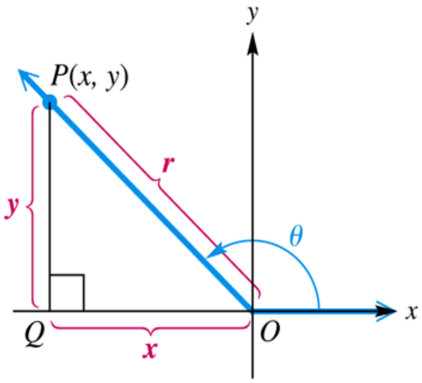
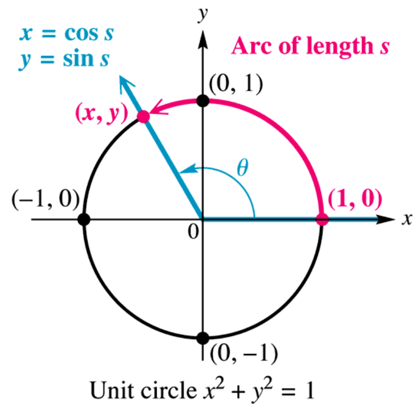
Section 5.1: Fundamental Trigonometric Identities

I. Identities

- A **mathematical identity** is an equation that is satisfied by *every value* in the domain of its variable.

- We have already developed several *fundamental identities* based on our triangle and unit circle definitions of the six trigonometric functions.





Fundamental Identities

Reciprocal Identities

$$\cot\theta = \frac{1}{\tan\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \csc\theta = \frac{1}{\sin\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \quad \tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Negative-Angle Identities

$$\sin(-\theta) = -\sin\theta \quad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \quad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \quad \cot(-\theta) = -\cot\theta$$

Note
In trigonometric identities, θ can be an angle in degrees, a real number, or a variable.

- These identities can be used both to find **numerical values** and **symbolically** to simplify expressions.

II. Using the Fundamental Identities

Example 1 (Finding Trigonometric Function Values Given One Value and the Quadrant): If $\cos\theta = \frac{5}{8}$ and θ is in quadrant IV, find each function value.

a) $\sin\theta$	**b)** $\tan\theta$
c) $\sec(-\theta)$	**d)** $\csc(-\theta)$

Example 2 (Expressing One Function in Terms of Another): Express $\tan\theta$ in terms of $\cos\theta$.

- Since the sine and cosine function are the two fundamental trigonometric functions, it is very common to *rewrite expressions* entirely in terms of them.

Example 3 (Rewriting an Expression in Terms of Sine and Cosine): Rewrite the expression

$$\frac{1+\tan^2\theta}{1-\sec^2\theta}$$

in terms of $\sin\theta$ and $\cos\theta$. Then simplify the expression so that no quotient appears.