

Section 5.2: Verifying Trigonometric Identities

I. Strategies

- Remember that a **mathematical identity** is an equation that is satisfied by *every value* in the domain of its variable. Sometimes these identities need proof.
- One skill required for more advanced work in mathematics is the ability to *use identities to rewrite expressions* in an alternate and more palatable form.
- There is **no substitute for experience** when it comes to verifying identities. However, here are some *helpful hints* to guide you along the way.

Hints for Verifying Identities

1. ***Learn the fundamental identities.*** Whenever you see either side of a fundamental identity, the other side should come to mind. ***Also, be aware of equivalent forms of the fundamental identities.***

Example: The Pythagorean Identities

2. ***Try to rewrite the more complicated side*** of the equation so that it is identical to the simpler side.
3. ***It is sometimes helpful to express all trigonometric functions in the equation in terms of sine and cosine*** and then simplify the result.
4. ***Usually, any factoring or indicated algebraic operations should be performed.***

Example: $\sin^2 x + 2\sin x + 1$ can be factor.

5. *As you select substitutions, keep in mind the side you are not changing, because it represents your goal.*
6. If an expression contains $1 + \sin x$, $1 - \sin x$, $1 - \cos x$, or $1 + \cos x$ *multiply both numerator and denominator by the conjugate.*

Caution

The procedure for verifying identities is not the same as that of solving equations.

Techniques used in solving equations, such as adding the same term to each side, and multiplying each side by the same term, should not be used when working with identities.

II. Verifying Identities by Working with One Side

- To avoid the temptation to use algebraic properties of equations to verify identities, *one strategy is to work with only one side and rewrite it to match the other side.*

Example 1: Verify that the following equation is an identity.

$$\sec x(\sin x + \cos x) = 1 + \tan x$$

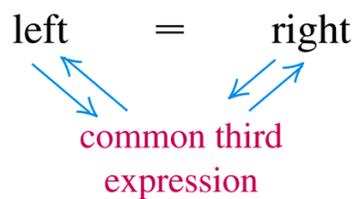
Example 2: Verify that $\cot^2 x(\tan^2 x + 1) = \csc^2 x$ is an identity.

Example 3: Verify that $\frac{\tan^2 s}{\sec^2 s} = (1 + \cos s)(1 - \cos s)$ is an identity.

Example 4: Verify that $\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$ is an identity.

III. Verifying Identities by Working with Both Sides

- If both sides of an identity appear to be equally complex, the identity can be verified by working independently on each side until they are changed into a common third result.
- *Each step, on each side, must be reversible.*



Example 5: Verify that $\frac{\cot x - \csc x}{\cot x + \csc x} = \frac{1 - 2\cos x + \cos^2 x}{-\sin^2 x}$ is an identity.