Section 5.4: Sum and Difference Identities for Sine and Tangent

I. Sum and Difference Identities for Sine

Derivation: We can use the cosine sum and difference identities to derive similar **identities for sine**.

Sine of a Sum or Difference

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

II. Sum and Difference Identities for Tangent

Derivation: We can use the sine and cosine sum and difference identities to derive similar **identities for tangent**.

Sine of a Sum or Difference

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

III. Applying the Sum and Difference Identities

Example 1: **a)** Find the *exact* value of sin 75°.

b) Find the *exact* value of $\tan\frac{7\pi}{12}$.

c) Find the *exact* value of

$$\sin 40^\circ \cos 160^\circ - \cos 40^\circ \sin 160^\circ.$$

Example 2: Write each function as an expression involving functions of θ.

a) $\sin(30° + \theta)$

b) $\tan(45° - \theta)$

c) $\sin(180° - \theta)$

Example 4: Verify that the equation is an identity.

$$\sin\left(\frac{\pi}{6} + \theta\right) + \cos\left(\frac{\pi}{3} + \theta\right) = \cos\theta$$

Example 3: Suppose that A and B are angles in standard position with $\sin A = \frac{4}{5}, \frac{\pi}{2} < A < \pi$, and $\cos B = -\frac{5}{13}, \pi < \mathrm{B} < \frac{3\pi}{2}$. Find each of the following.

(a) $\sin(A + B)$

(b) $\tan(A + B)$

(c) the quadrant of $A + B$