Section 5.5: Double-Angle Identities

I. Double-Angle Identities for Sine, Cosine, and Tangent

Derivation 1: We can use the sine sum identity to derive what's known as **the double-angle identity for sine.**

Derivation 2: Similarly, we can use the cosine sum and difference identities to derive what's known as **the double-angle identities for cosine.** There are *3 such identities*, all of them frequently used.

Derivation 3 (You try this one!): Finally, we can use the tangent sum identity to derive what's known as **the double-angle identity for tangent.**

Double-Angle Identities

$\sin 2A = 2\sin A\cos A$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

II. Applying the Identities

Example 1 (Finding Function Values of 2*θ* Given Information About *θ*): Given that $\sin\theta = \dfrac{8}{17}$ and $\cos\theta < 0$, find the values of $\sin 2\theta, \cos 2\theta$, and $\tan 2\theta$.

Example 2 (Practice): Find the values of $\sin 2\theta, \cos 2\theta$, and $\tan 2\theta$ given that $\cos 2\theta = -\frac{12}{13}$ and that $180^\circ < \theta < 270^\circ$.

Example 3 (Verifying a Double-Angle Identity): Verify that

$$\cos^4 x - \sin^4 x = \cos 2x$$

is an identity.

Example 4 (Simplifying an Expression): Simplify each expression.

a) $2\cos^2 5x - 1$

b) $\sin 165^\circ \cos 165^\circ$

Example 5(Deriving an Identity): Write $\cos(3x)$ in terms of $\cos x$.

- **Side-Note:** Rewriting a trigonometric function so that the argument is only x and not $2x, 3x, \ldots$ is a very important skill.

III. Product-to-Sum and Sum-to-Product Identities

- I will give you these on any exam if you need them!

Identities

$$\cos A\cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$$

$$\sin A\sin B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$$

$$\sin A\cos B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$$

$$\cos A\sin B = \frac{1}{2}[\sin(A+B)-\sin(A-B)]$$

$$\sin A+\sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A-\sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A+\cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A-\cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Example 6: Write $\cos 3x+\cos 7x$ as the product of two functions

Example 7: Write $6\sin 40^\circ \sin 15^\circ$ as the sum or difference of two functions.