7.3 The Hyperbola

Objectives

A. Understand the Definition of a Hyperbola
B. Write Equations of Hyperbolas
C. Graph Hyperbolas

A. Understand the Definition of a Hyperbola

The Hyperbola

- A **hyperbola** is the set of all points $P$ in a plane such that the absolute value of the difference of the distances from point $P$ to two other points, $F$ and $F'$, in the plane is a positive constant.

- Note that $d(F_1P) - d(F_2P)$ will be a positive ______________.

B. Write the Equation of a Hyperbola

Hyperbola: Foci on the $x$-Axis, Center at $(0, 0)$

- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Hyperbola: Foci on the $y$-Axis, Center at $(0, 0)$

- $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

All Hyperbolas:

- Each of the endpoints $V$ and $V'$ of the transverse axis is called a ______________.
- $c^2 = a^2 + b^2$
- $a$ is not necessarily larger than $b$ for hyperbolas!
**Equation of the Hyperbola with Center at \((h,k)\):**

- \(c^2 = a^2 + b^2\)
- \(a\)=distance from center to vertex AND half the length of a side of the fundamental rectangle
- \(b\)= half the length of the “other” side of the fundamental rectangle
- \(c\)=distance from the center to the focus

<table>
<thead>
<tr>
<th>Horizontal Transverse Axis:</th>
<th>Vertical Transverse Axis:</th>
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<tr>
<td>[ \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 ]</td>
<td>[ \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1 ]</td>
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**Example:** Find the equation of each hyperbola with the given properties:

1) With vertices (0,4) and (0,-4) and a focus of (0,5)

2) With vertices (3,1) and (-3,1) and a focus of (5,1)

3) With vertices (3,-3) and (3,3) and a focus of (3,5)

4) Foci (5,0) and (-5,0) and \( \frac{c}{a} = \frac{5}{4} \)
C. Graph Hyperbolas

Method for Graphing Hyperbolas

1. Write the equation in standard form.
   a. Factor out the coefficients for $x$ and $y$.
   b. Complete the square for $x$ and $y$ (careful w/ adding the same amount to both sides)
   c. Divide to set the equation equal to _____.

2. Find the center, $a$, and $b$.

3. Graph the center, the vertices, the fundamental rectangle (using $a$ and $b$), and the asymptotes, and then sketch the hyperbola.

Examples: Graph each hyperbola with the given equation:

1) \[ \frac{y^2}{9} - \frac{(x+1)^2}{25} = 1 \]

2) \[ \frac{(x+3)^2}{36} - \frac{(y-1)^2}{16} = 1 \]

3) \[ 9x^2 - 4y^2 - 18x - 16y = 43 \]
4) \( y^2 - 4x^2 + 32x + 6y = 59 \)

**Example:** Find the area of the fundamental rectangle of the hyperbola given by 
\( x^2 - y^2 - 4x - 6y = 6 \). Hint: sketch the hyperbola first.