Multiplying and Dividing Rational Expressions

Multiplying and dividing rational expressions follows the same format as multiplying and dividing fractions, the only difference is that you must factor the rational expressions before simplifying the common factors.

**Multiplication**

You multiply fractions by multiplying across: \( \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \). If possible, you can simplify before multiplying – remember you must simplify in both the numerator and denominator. For example: \( \frac{a}{b} \cdot \frac{b}{c} = \frac{a}{c} \).

**Example 1:** Multiply the rational expressions, be sure the answer is simplified:

\[
\frac{2x-6}{x+4} \cdot \frac{3x+12}{5x-15}
\]

**Solution:** The first step is to factor everything completely, then get rid of the common factors between the numerator and denominator.

\[
\frac{2(x-3)}{x+4} \cdot \frac{3(x+4)}{5(x-3)}
\]

\[
\frac{2(x-3)}{x+4} \cdot \frac{3(x+4)}{5(x-3)} = \frac{6}{5}
\]

**Example 2:** Multiply the rational expressions, be sure the answer is simplified:

\[
\frac{x^2-9}{x^2+7x+12} \cdot \frac{x-1}{x^2+2x-15}
\]

**Solution:**

\[
\frac{x^2-9}{x^2+7x+12} \cdot \frac{x-1}{x^2+2x-15} = \frac{(x+3)(x-3)}{(x+3)(x+4)} \cdot \frac{x-1}{(x-3)(x+5)}
\]

\[
\frac{(x+3)(x-3)}{(x+3)(x+4)} \cdot \frac{x-1}{(x-3)(x+5)} = \frac{x-1}{(x+4)(x+5)}
\]

Note: It is easiest (and best) to leave the answer in factored form – it is not necessary to multiply out the denominator.
Division

Division of fractions is the same as multiplying the first fraction by the reciprocal of the second fraction (always take the reciprocal of the fraction to the right of the division symbol).

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}
\]

Example 3: Divide – be sure the answer is simplified: \[\frac{x^2 + 3x - 40}{x^2 - 25} \div \frac{x^2 + 9x + 8}{x^2 - 3x - 4}\]

Solution: The first step is to change the division problem to a multiplication problem. The next step is to factor everything and multiply.

\[
\frac{x^2 + 3x - 40}{x^2 - 25} \cdot \frac{x^2 - 3x - 4}{x^2 + 9x + 8} = \frac{(x+8)(x-5)}{(x+5)(x-5)} \cdot \frac{(x-4)(x+1)}{(x+8)(x+1)}
\]

\[
\frac{(x+8)}{(x+5)} \cdot \frac{(x-4)}{(x+8)} = \frac{x-4}{x+5}
\]

This section heavily depends on your factoring ability. Be sure to review your factoring worksheets, including how to factor the difference of squares and the sum/difference of cubes.
Practice Problems

Multiply and divide the rational expressions – be sure all answers are simplified completely.

1. \(\frac{x^3 - 4x - 21}{2x+1} \cdot \frac{2x^3 + x^2}{x^2 - 7x}\)

2. \(\frac{5x-5 \cdot x+7}{x^2 - 49} \cdot \frac{x}{x^2 + 1}\)

3. \(\frac{x}{8-4x} \div \frac{x^2}{6-3x}\)

4. \(\frac{x^2 - 49}{7x - 49} \div \frac{x^2 + 13x + 42}{x^2 + 6x^2}\)

5. \(\frac{x^3 - 2x - 15}{16 - 9x^2} \cdot \frac{6x - 8}{2x^2 - 9x - 5}\)

6. \(\frac{x^2 + 12x + 27}{x^2 + 6x - 7} \div \frac{x^2 + 16x + 63}{x^2 - 13x + 12}\)

7. \(\frac{3x^2 + x - 10}{2x^2 + 3x - 2} \div \frac{3x^2 + 7x - 20}{x^2 + x - 12}\)

8. \(\frac{x^3 - 8}{x^2 - 4} \cdot \frac{x^2 + 5x + 6}{x^2 - 9}\)

9. \(\frac{24x^2 + 44x - 28}{2x^2 - 7x + 3} \div \frac{9x^3 - 49}{x^2 + 2x - 15}\)

10. \(\frac{10x^3 + 29x + 10}{2x^2 - 9x - 35} \cdot \frac{6x^2 + 7x - 20}{15x^2 - 14x - 8}\)

Remember order of operations for the last two problems:

11. \(\frac{2x-1}{x+1} \div \frac{6x - 3}{3} \cdot \frac{4x + 4}{x}\)

12. \(\frac{2x^2 - 5x - 3}{25x^2 + 15x + 2} \div \frac{2x^2 + 7x + 3}{5x^2 - 18x - 8} \cdot \frac{x - 4}{x^2 - 4x + 3}\)