Angular Measurements

1.0 Purpose:
To determine the angular size of different parts of one's body, then use these values to measure/calculate:
a) the angular distance to an object in the sky
b) the angular size of distant objects
c) the distance to the Toll Road on the horizon
then compare these measurements to the accepted/theoretical values in Table 2.

2a. Equipment:
   Meter stick and Calculator and Body Parts

2b. Theory:
An arc length is a segment of arc along a circle like that labeled “s” in figures (1a) and (1b) below.

In the figures & equations
\[ r = \text{arc radius} \]
\[ \theta = \text{angle/angular size} \]
\[ s = \text{arc length} \]
\[ D = \text{chord length} \]

Figure (1a)  Figure (1b)

Fill in the blanks next to the bullets below.
- What is the circumference of a circle (i.e. the distance all of the way around the circle) in terms of the radius of the circle, \( r \), shown in the figures above. ______
- Recalling that angles are measured in both the units of degrees and radians, how many radians are swept through when going once around a circle (i.e. 360 degrees). ______ radians
- Do you see a relationship between your first and second answers? Explain any similarities and differences here: _______________________________
- Realize that if you knew the answer to the first question then you already know the arc-length formula that will be used in this lab experiment. Now, instead of finding the arc length of an entire circle which is called the circumference, the length of a smaller arc of the circle will be studied, such as the one labeled “s” in Figure (1). Using your answers to the questions above, write down the length of the arc, \( s \), in terms of the angle \( \theta \) and radius \( r \) identified in Figures (1a) & (1b), in the space below.
Arc-Length Formula: \( s = \) ______
**Hint:** Notice that the arc-length of a whole circle (the circumference) is $2\pi r$, where the angle swept out when going in a full circle is $2\pi$ radians for a circle of radius $r$. So it appears that arc-length, $s$, is (angle)(radius) yielding Equation (1) below.

**Arc-Length Formula:**

$$s = r\theta$$  \hspace{0.5cm} \text{Equation (1)}

For small angles, $\theta$, of ~14 degrees or less, arc-length equals chord length, i.e. $s = D$, and the Arc-Length Formula becomes the...

**Small Angle Formula:**

$$D = r\theta$$  \hspace{0.5cm} \text{Equation (2)}

**CONVERSION of $\theta$:**
The equations above have $\theta$ in radians. Multiply the angle $\theta$ by 57.29 to convert it from radians to degrees. Multiply $\theta$ by $1/57.29$ to convert it from degrees to radians.

The diagram below shows why small angles are needed in order for $s$ to equal $D$, and make Equation (2) valid.

- Which picture below, Figure (2) or (3), has the smaller angle, $\theta$, in the pointed region?  
  If you find it difficult to answer this question, it might help to extend the arcs around to make a semi circle or a circle and then look at the angles compared to 180 degrees or 360 degrees.

- Which figure below has the chord and arc lengths nearly equal, i.e. $s \approx D$? Is this the figure with the bigger or smaller angle, $\theta$?

- Therefore, if it is assumed that $s \approx D$ then small angles, $\theta$, must be used, which means $\theta$ should be less than or equal to ~14 degrees.

![Figure (2)](image1.png)  \hspace{1cm} ![Figure (3)](image2.png)

In this lab, the **Small Angle Formula**, Equation (2), will be rearranged and used to determine the angular size of ones body parts (index finger, thumb, fist, hand) which will then be used in determining: 1) the **angular size/distance** to objects, such as Polaris and the Toll Road on the horizon and 2) the **distance** to the Toll Road on the horizon.
When calculating the angular size, $\theta$, of various objects or the angular distance to objects, Equation (2) will be rearranged to this form to isolate $\theta$:

$$\theta = \frac{D}{r} \quad \text{Equation (3)}$$

When calculating the distance to an object if its width is known and its angular size can be measured (e.g. the Toll Road on the horizon), Equation (2) will be rearranged to this form to isolate $r$:

$$r = \frac{D}{\theta} \quad \text{Equation (4)}$$

To determine angular size or distance of an object, $\theta_{\text{object}}$, using number of body lengths, $n$, and the angular size of the body parts, $\theta_{\text{body parts}}$, from Table 1.

$$\theta_{\text{object}} = n \cdot \theta_{\text{body parts}}$$

or

angular size of object = (# body lengths)(angular size of body part)

Equation (5)

Percent Difference Formula:

$$\% \ \text{difference} = \left( \frac{\text{THEORETICAL}_{\text{value}} - \text{EXPERIMENTAL}_{\text{value}}}{\text{THEORETICAL}_{\text{value}}} \right) \times 100 \quad \text{Equation (6)}$$

2.0 Procedure:

1. Review the arc length formula for small angles then obtain a meterstick and calculator.
2. To determine your dominant eye, the eye you want to use for determining angular distance, follow the process below:
   a) Focus on a distant object and cover it with your right finger while your arm is fully extended straight out in front of you and both eyes are open.
   b) Now as you close your right eye, notice if your finger appears to move. If it appears stationary then your right eye is your dominant eye.
   c) Otherwise, open the right eye and close the left eye and try the same process with the left eye. Whichever eye appears to keep your finger stationary is the dominant eye which you want to use for determining angular distance.
3. Extend your arm reaching straight out and slightly upward, holding up the body part that you are going to measure the angular distance of. For example starting with the index finger, measure the distance (in mm or cm) from your eye to your index finger “$r$” on your extended arm. Measure the width of your index finger and call this “D”.
4. Record this information in the first table.
5. Do the same thing for all other body parts listed in Table 1. Some body parts are shown below.
6. Calculate each body part’s angular size using Equation (3).

7. After calculating the angular size for each body part, go to the roof of the Math/Science building and measure (using body parts)
(a) the angular distance from the North star to the horizon
(b) the angular width/size of the 73 Toll Road (at the horizon)
Be sure to record all values in Table 2.

8. Now calculate the experimental distance, \( r \), to the horizon of the Toll Road assuming it has an actual width of 200 ft. Use Equation (4) and your experimentally measured angular width, \( \theta \), of the Toll Road at the horizon that was obtained in step 7. Record this in Table 2.

9. **Calculate the actual/theoretical value for the Toll Road’s angular width at the horizon by using Equation (3) and plugging in the actual distance from you to the horizon of 1 mile = 5280 ft and its actual width of 200 ft. Record this value for \( \theta \) in Table 2. below.

### 3.0 Data:

<table>
<thead>
<tr>
<th>Body Part</th>
<th>Width of body part, ( D ) (cm)</th>
<th>Distance to body part from eye, ( r ) (cm)</th>
<th>Angular Size of body part, ( \theta ) [use Equation (3) &amp; multiply by 57.29] (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index finger</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thumb</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fist</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hand</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1.

<table>
<thead>
<tr>
<th>Quantity Measured</th>
<th>How many body lengths? (e.g. 1.5 hands)</th>
<th>Experimental Angular Size or Distance ( \theta ) (degrees) [use Equation (5)]</th>
<th>Theoretical or Accepted Angular Size or Distance, ( \theta ) (degrees) [use Equation (4) &amp; * to left]</th>
<th>Experimental Distance, ( r ) (feet)</th>
<th>Theoretical or Accepted Distance, ( r ) (feet)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Star to ground</td>
<td></td>
<td>33.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Toll Road at horizon (*actual width is ( D = 200 ) ft)</td>
<td></td>
<td>(see ** above)</td>
<td>5280 ft = 1 mile</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.
4.0 Results & Analysis:

[NOTE: You must include your own sample calculations in your lab write-up, not those shown below, which are simply an example.]

- Sample calculation of Angular Size of my hand using Equation (3)
  \[ r\theta = D \]
  \[ \theta = \frac{D}{r} \]
  \[ \theta = \frac{18 \text{ cm}}{55 \text{ cm}} \times 57.29 \]
  \[ \theta = 18.7^\circ \] is angular size of my hand when oriented as shown in picture above.

- Sample calculation of the distance from the North Star to the ground using Equation (5):
  \[ \theta_{\text{object}} = n \cdot \theta_{\text{body parts}} \]
  \[ \theta = 2 \text{hands} \times \left( \frac{18.7^\circ}{1 \text{ hand}} \right) \]
  \[ \theta = 37.4^\circ \] degrees is angular distance from horizon to North star

- Sample calculation of distance to the horizon of the 73 toll road. Assuming the width of the Toll Road is 200 m (actual width), using the angular size of the Toll Road which I found using my thumbs and using Equation (5).
  \[ r\theta = D \]
  \[ r = \frac{D}{\theta} \]
  \[ r = \frac{200 \text{ ft}}{3.1^\circ / 57.29} \] (where \( \theta \) is multiplied by 1/57.29 to convert degrees to radians)
  \[ r = 3677 \text{ ft} \] is the experimental distance to the horizon of toll road from the MSE roof

Perform all calculations, filling in the tables and use Equation (6) to determine the Percent Differences in Table 2.

5.0 Conclusion

Write up a detailed conclusion following the Lab Report Write-Up Guidelines.