Solving Equations Part I

Recall: An equation is a statement that two algebraic expressions are **equal**. ALL equations have an **equal sign.**

Example 1:

Consider the equation

$$\mathbf{x - 3 = 7}.$$

To solve for $\mathbf{x}$ by inspection, we ask, **What number do you subtract $\mathbf{3}$ from to get $\mathbf{7}$**?

Since $10 - 3 = 7$, $\mathbf{x = 10}$. We can solve this equation **algebraically**. Keep in mind that our ultimate goal is to have $\mathbf{x}$ by itself on one side of the equal sign.

In the equation

$$\mathbf{x - 3 = 7}$$

we need to change the left hand side from $\mathbf{x - 3}$, we get

$$\mathbf{x - 3 + 3 = x + 0 = x.}$$

In order to keep both sides of the equation **equal**, we must also **add $\mathbf{3}$** to the right hand side.

$$\begin{array}{l} \mathbf{x - 3 = \ \ 7} \\ \underline{\ \mathbf{+3 \quad +3}\ } \\ \mathrm{x} = 10 \end{array} \qquad \text{[Now } \mathbf{x} \text{ is alone.]}$$

So, $\mathbf{x = 10}$ is the solution.

Example 2:

Consider the equation:

$$\mathbf{x + 3 = 7}$$

By inspection, $\mathbf{x} =$ ________.

To solve this equation **algebraically**, we must **subtract $\mathbf{3}$** from both sides of the equation in order to **isolate the variable**.

$$\begin{array}{rcr} \mathbf{x + 3} & = & \mathbf{7} \\ \underline{\mathbf{-3}} & & \underline{\mathbf{-3}} \\ \mathbf{x} & = & \mathbf{4} \end{array}$$

So $\mathbf{x = 4}$ is the solution.

This method of adding $\mathbf{3}$ to both sides in example 1 and subtracting $\mathbf{3}$ from both sides in example 2 is called the **Addition Property of Equality**.

The **Addition Property of Equality.** states that we can add the same value to both sides of an equation without changing the solution.

> **NOTE:** Recall that subtracting a number is the same as **adding** the opposite of that number. This implies that we can also **subtract** the same value from both sides of an equation without changing the

Example 3:

Consider the equation:

$$\mathbf{4x = 20}$$

To solve for $\mathbf{x}$ by inspection, we ask, **What number do you multiply by $\mathbf{4}$ to get $\mathbf{20}$**?

By inspection $\mathbf{x =}$ ______.

We can solve this equation **algebraically**. Keep in mind that our ultimate goal is to have $\mathbf{x}$ by itself on one side of the equal sign.

In the equation $\mathbf{4x = 20}$, we need to change the left hand side to just $\mathbf{x}$. If we **divide by $\mathbf{4}$** we get $\mathbf{\frac{4x}{4} = x}$. In order to keep both sides of the equation **equal**, we must also **divide by $\mathbf{4}$** on the right hand side,

$$\mathbf{\frac{4x}{4} = \frac{20}{4}}$$

$$\mathbf{x = \frac{20}{4} = 5}$$

So $\mathbf{x = 5}$ is the solution.

Example 4:

Consider the equation.

$$\frac{\mathbf{x}}{\mathbf{3}} = \mathbf{4}$$

By inspection, $\mathbf{x} =$ ______.

To solve this equation **algebraically**, we must **multiply** both sides by $\mathbf{3}$ in order to **isolate the variable.**

$$\mathbf{3} \cdot \frac{\mathbf{x}}{\mathbf{3}} = \mathbf{3} \cdot \mathbf{4}$$

$\mathbf{x} =$ _______.

This method of dividing by $\mathbf{4}$ in example 3 and multiplying by $\mathbf{3}$ in example 4 is called the **Multiplication Property of Equality.**

The **Multiplication Property of Equality** states that we can multiply both sides of an equation by the same value without changing the solution.

NOTE: Recall that dividing by a number is the same as **multiplying** by its reciprocal. This implies that we can also **divide** both sides of an equation by the same value without changing the

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Practice Problems

Solve each equation algebraically. Check your answers by inspection.

1. $\mathbf{x - 5 = 12}$

2. $\mathbf{x - 3 = -4}$

3. $\mathbf{x + 5 = 12}$

4. $\mathbf{x + 5 = -4}$

5. $\mathbf{\frac{x}{7} = 2}$

6. $\mathbf{9x = -27}$