Slope of a Line

Consider the equation

$$y = 2x - 4$$

We can make an x, y chart to help graph the line represented by this equation.

x	y
1	-2
2	0
3	2
4	4

Do you notice a pattern between the change of y values and the change of x values?

This pattern is referred to as the slope of the line.

The slope of a line is the ratio of the change in y (vertical change) to the change in x (horizontal change). We use the letter m to represent slope.

$$\boxed{m = \frac{\text{change of } y}{\text{change of } x}}$$

Let’s look at:

$$y = 2x - 4$$

The $\boldsymbol{x}$ values increase by 1

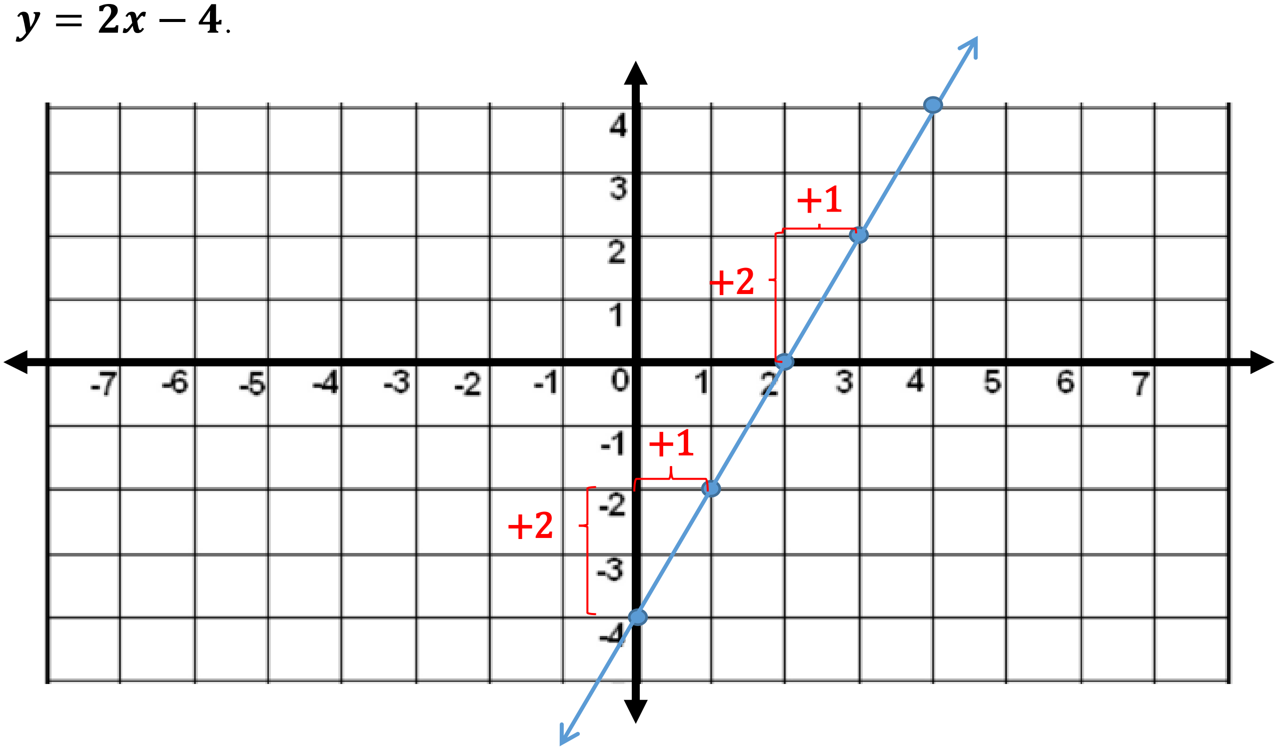
	x	y	
	1	-2	
$+1$			$+2$
	2	0	
$+1$			$+2$
	3	2	
$+1$			$+2$
	4	4	

The $\boldsymbol{y}$ values increase by 2

$$m = \frac{change\ of\ y}{change\ of\ x} = \frac{2}{1} = 2$$

therefore, $\boldsymbol{m = 2}$ for the line $\boldsymbol{y = 2x - 4}$.

We can use the slope to help graph the line represented by the equation $\boldsymbol{y = 2x - 4}$.

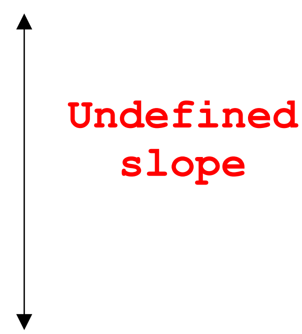


Looking left to right:

If the line is going up, the line has a **negative** slope

If the line is going down, the line has a **positive** slope

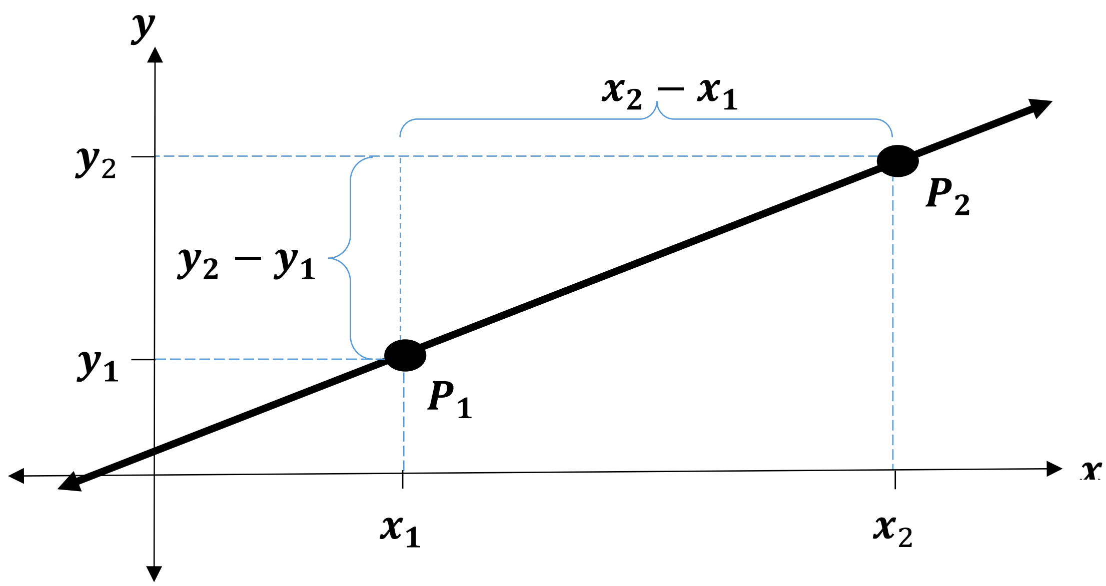
All horizontal lines have a slope equal to **zero**.



All vertical lines have an **undefined** slope.

Finding the slope of a line given two points on the line:

Let's call $\boldsymbol{P_1}$ (point $\mathbf{1}$) $(\boldsymbol{x_1}, \boldsymbol{y_1})$ and
$\boldsymbol{P}_2$ (point $\mathbf{2}$) $(\boldsymbol{x}_2, \boldsymbol{y}_2)$



$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1

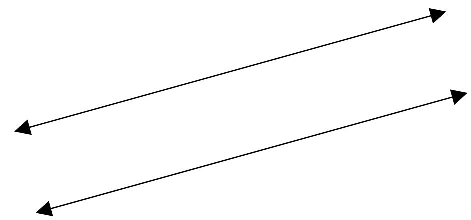
Find the slope of the line that passes through the points $(\mathbf{6}, \mathbf{2})$ and $(\mathbf{18}, \mathbf{8})$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 2}{18 - 6}$$

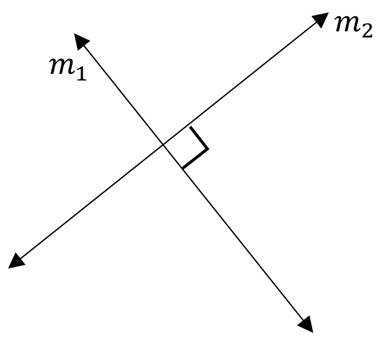
$$= \frac{\quad}{\quad} = \boxed{\frac{\quad}{\quad}}$$

*reduce

<u>Parallel Lines</u>:
Two lines are **parallel** if they never intersect. **Parallel lines have the same slope**.



<u>Perpendicular Lines:</u> Two lines are **perpendicular** if and only if they intersect at a $\mathbf{90°}$ angle. **The slopes of two perpendicular lines are negative reciprocals of each other**.



$\boldsymbol{m_1}$	$\boldsymbol{m_2}$
$\mathbf{3}$	$-\dfrac{\mathbf{1}}{\mathbf{3}}$
$-\dfrac{\mathbf{2}}{\mathbf{3}}$	$\dfrac{\mathbf{3}}{\mathbf{2}}$
$\mathbf{0}$	$\boldsymbol{Undefined}$

Finding the slope using the equation of a line:

> **<u>Recall:</u>** Equation of a line
> $\boldsymbol{y = mx + b}$

Step 1: Solve the equation for $\boldsymbol{y}$

Step 2: identify the coefficient of $\boldsymbol{x}$ – this is the slope of the line.

Example 2

Find the slope of the line $\boldsymbol{2x+y=8}$

Step 1:

$$\begin{array}{rcl} \boldsymbol{2x+y} & \boldsymbol{=} & \boldsymbol{8} \\ \boldsymbol{-2x} & & \boldsymbol{-\,2x} \\ \hline \boldsymbol{y} & \boldsymbol{=} & \boldsymbol{-2x+8} \end{array}$$

<u>NOTE</u>: we prefer to see the $\boldsymbol{x}$ before the constant

Step 2: The coefficient of $\boldsymbol{x}$ is $\boldsymbol{-2}$

Therefore, the slope of the line $2x+y=8$ is $\boldsymbol{-2}$.

<u>NOTE:</u> Any line parallel to the line $\boldsymbol{2x+y=8}$ has a slope of $\boldsymbol{-2}$ ands any line perpendicular to the line has a slope of $\boldsymbol{\frac{1}{2}}$.

Example 3

Identify the lines as being parallel, perpendicular or neither:

$$x + y = 4$$
$$3x + 3y = 6$$

First we find the slope of each line.

Step 1: Solve for $\boldsymbol{y}$ (for both Line 1 and Line 2)

Line 1:

$$\begin{array}{rcl} x + y &=& 4 \\ \underline{-x \qquad} && \underline{-x \quad} \\ y &=& -x + 4 \end{array}$$

Slope of line 1: $m_1 = -1$

Line 2:

$$\begin{array}{rcl} 3x + 3y &=& 6 \\ \underline{-3x \qquad} && \underline{-3x \quad} \\ \frac{3y}{3} &=& \frac{-3x}{3} + \frac{6}{3} \\ y &=& -x + 2 \end{array}$$

Slope of line 2: $m_2 = -1$

$\boxed{\text{Therefore, these two lines are } \textbf{parallel}}$

Slope of a Line

Practice Problems

1. Find the slope of the line that passes through the points $(5, 7)$ and $(-1, 3)$.

2. Match each equation with its corresponding graph:

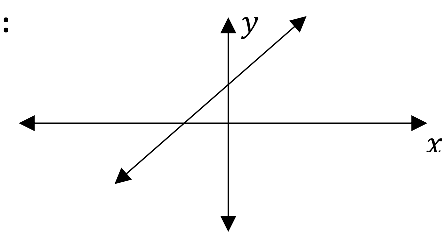
a) $x + y = 1$

b) $-x + y = 1$

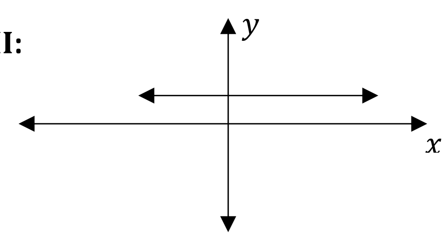
c) $y = 1$

d) $x = 1$

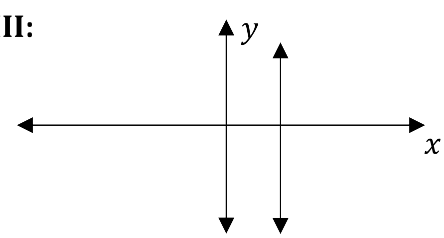
I:



II:



III:



IV: