Special Factoring Rules

Difference of Two Squares

Quadratics of the form $\boldsymbol{a^2 - b^2}$ are called a difference of two squares and can be factored using the formula:

$$\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$$

Example 1:

Factor each quadratic completely.

a.) $\boldsymbol{x^2 - 9}$

b.) $\boldsymbol{x^2 - 16}$

c.) $\boldsymbol{2x^2 - 8}$

Since both terms divide evenly by $\mathbf{2}$, we factor out the $\mathbf{2}$:

$$2x^2 - 8 = \mathbf{2}(\boldsymbol{x^2 - 4})$$

The resulting quadratic is a difference of two squares, therefore we can factor further:

$$2(x^2 - 4) = \mathbf{2}(\boldsymbol{x + 2})(\boldsymbol{x - 2})$$

NOTE: a **sum** of two squares **CANNOT** be factored! It is considered to be **PRIME**.

Sum and Difference of Two Cubes:

The sum of two cubes is a polynomial of the form $\boldsymbol{a^3 + b^3}$ and can be factored using the formula:

$$\boldsymbol{a^3 + b^3 = (a + b)(a^2 - ab + b^2)}$$

The difference of two cubes is a polynomial of the form $\boldsymbol{a^3 - b^3}$ and can be factored using the formula:

$$\boldsymbol{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$$

Example 2:

Factor each polynomial.

a.) $\boldsymbol{x^3 - 8}$

$=$

b.) $\boldsymbol{x^3 + 1}$

$=$

c.) $\boldsymbol{3x^3 - 81}$

$=$

Since both terms divide evenly by $\mathbf{3}$, we factor out the $\mathbf{3}$ first:

$\boldsymbol{3x^3 - 81 = 3(\quad) = 3(\quad)(\quad)}$

Perfect Squares

A perfect square trinomial is a trinomial of the form $\boldsymbol{a^2 + 2ab + b^2}$ or $\boldsymbol{a^2 - 2ab + b^2}$ and can be factored using the formulas:

$$\boldsymbol{a^2 + 2ab + b^2 = (a + b)^2}$$

$$\boldsymbol{a^2 - 2ab + b^2 = (a - b)^2}$$

Example 3:

Factor each trinomial:

a.) $\boldsymbol{x^2 + 6x + 9}$

b.) $\boldsymbol{x^2 - 12x + 36}$

c.) $\boldsymbol{2x^2 + 20x + 50}$

NOTE:
All trinomials, even "special" ones, can be factored using the abc method!!!

Special Factoring Rules — Practice Problems

Factor each polynomial if possible:

1. $\mathbf{9x^2 - 25}$

2. $\boldsymbol{a^2 + b^2}$

3. $\mathbf{2x^3 + 16}$

4. $\mathbf{4x^2 + 32x + 16}$

5. $\mathbf{5x^2 - 10x + 5}$