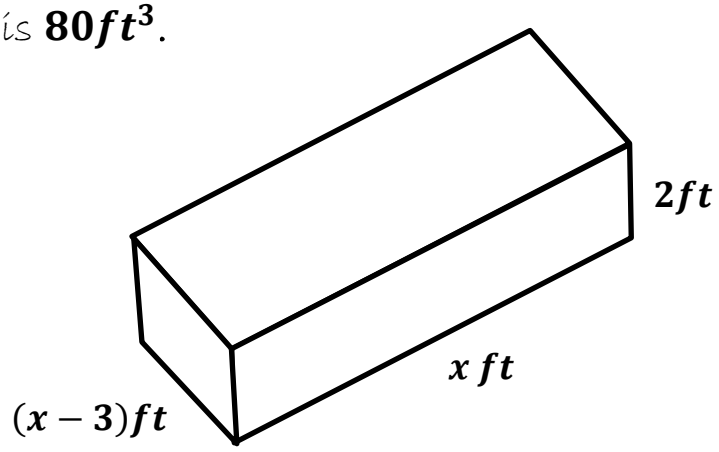
Applications of Quadratic Equations

Geometry Problems

Example 1:

Find the length and width of the box below, given that the volume of the box is $\mathbf{80}\boldsymbol{ft^3}$.



Recall: The formula for the volume of a box (rectangle) is:

$$\boldsymbol{Volume = (length)x(width)x(height)}$$

$$\boldsymbol{V = l \cdot w \cdot h}$$

So we get

$$\mathbf{80 = (x)(x - 3)(2)}$$

Now we solve for $\boldsymbol{x}$:

$$\mathbf{80 = (2)(x)(x - 3)}$$

To simplify, we can divide both sides by $\mathbf{2}$.

$$\frac{80}{2} = \frac{(2)(x)(x-3)}{2}$$

$$\mathbf{40 = x(x - 3)} \qquad \text{Distribute.}$$

$$\mathbf{40 = x^2 - 3x} \qquad \text{Set equation equal to zero.}$$

$$\mathbf{x^2 - 3x - 40 = 0}$$

$$x^2 - 3x - 40 = 0$$

Using the abc method, we can factor:

$a = 1$
$b = -3$
$c = -40$

This means we can use the shortcut! ☺

$a \cdot c = -40$ Sum(we want -3)

1	-40	-39
2	-20	-18
4	8	-6
8	-5	3
-8	5	-3

So we get

$$(x - 8)(x + 5) = 0$$

$x - 8 = 0$ $\quad$ $x + 5 = 0$ ← Set each factor equal to 0, solve for x.

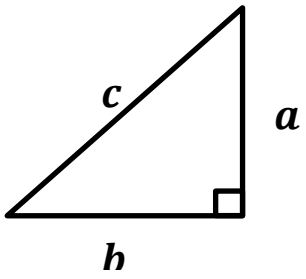
$+8 \quad +8$ $\quad$ $-5 \quad -5$

$x = 8$ $\quad$ $x = -5$

Which value of x do we choose? Since it is impossible to have a box with a length of $-5\ ft$, we choose $x = 8$.

Therefore, the length of the box is $8\ ft$ and the width of the box is $5\ ft$ $(8 - 3 = 5)$.

Pythagorean Theorem:



In any right triangle, the sum of the squares of the sides is equal to the square of the hypotenuse.

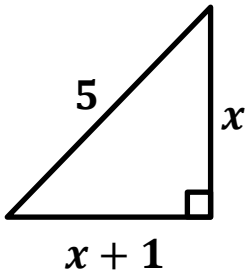
The formula is :

$$a^2 + b^2 = c^2$$

We will use this formula in the following example.

Example 2:

Find the side lengths of the triangle below:



Following the formula, we get:

$$x^2 + (x+1)^2 = 5^2$$

Now we solve for x.

$$x^2 + (x+1)(x+1) = 25$$
$$x^2 + x^2 + x + x + 1 = 25$$
$$2x^2 + 2x + 1 = 25$$ Set equation to zero.
$$-25 \quad -25$$
$$2x^2 + 2x - 24 = 0$$ Divide both sides by 2 to simplify.
$$\frac{}{2} \quad \frac{}{2}$$
$$x^2 + x - 12 = 0$$

Now we use the $\boldsymbol{abc}$ method to factor the trinomial:

$$\boldsymbol{x^2 + x - 12 = 0}$$

$$\boldsymbol{a = 1}$$
$$\boldsymbol{b = 1}$$
$$\boldsymbol{c = -12}$$

$\boldsymbol{a \cdot c = -12}$		SUM (we want 1)
$\boldsymbol{1}$	$\boldsymbol{-12}$	$\boldsymbol{-11}$
$\boldsymbol{2}$	$\boldsymbol{-6}$	$\boldsymbol{-4}$
$\boldsymbol{4}$	$\boldsymbol{-3}$	$\boldsymbol{1}$

Since $\boldsymbol{a = 1}$, we can use the shortcut:

$$\boldsymbol{(x + 4)(x - 3) = 0}$$ Set each factor to zero.

$\boldsymbol{x + 4 = 0}$ $\quad$ $\boldsymbol{x - 3 = 0}$ $\quad$ Solve for $\boldsymbol{x}$.

$\boldsymbol{-4 \quad -4}$ $\quad$ $\boldsymbol{+3 \quad +3}$

$\boldsymbol{x = -4}$ $\quad$ $\boldsymbol{x = 3}$

Which value do we choose?

Therefore the side lengths of the triangle are _____.

Applications of Quadratic Equations: Geometry Problems

Practice Problems

Find the length and width or the box below, given that the volume of the box is $\mathbf{60}\ \boldsymbol{ft}^3$.