# Applications of Quadratic Equations
## Integer Problems

**Recall**: Consecutive Integers are of the form:

$$x, x+1, x+2, \dots$$

Consecutive **odd** integers AND consecutive **even** integers are of the form:

$$x, x+2, x+4, \dots$$

Example 1:

Find two consecutive integers whose product is $\mathbf{11}$ more than their sum.

We must translate into math terms:

"two consecutive integers" → $x$, and $x+1$

"product" → multiplication

"is" → $=$ equal to

"$\mathbf{11}$ more than" → $+11$

"sum" → addition

So our equation is:

$$x(x+1) = x + (x+1) + 11$$

Now we solve for $x$:

Simplify each side first:

$$x^2 + x = 2x + 12$$

Set equation equal to zero:

$$\begin{array}{rcl} x^2 + x & = & 2x + 12 \\ \underline{-2x - 12} & & \underline{-2x - 12} \\ x^2 + x - 2x - 12 & = & 0 \end{array}$$

$$x^2 - x - 12 = 0$$

Factor: $\boldsymbol{x^2 - x - 12 = 0}$

$\boldsymbol{a = 1}$

$\boldsymbol{b = -1}$

$\boldsymbol{c = -12}$

| $\boldsymbol{a \cdot c = -12}$ | | SUM (we want $\boldsymbol{-1}$) |
|---|---|---|
| $\boldsymbol{1}$ | $\boldsymbol{-12}$ | $\boldsymbol{-11}$ |
| $\boldsymbol{2}$ | $\boldsymbol{-6}$ | $\boldsymbol{-4}$ |
| $\boldsymbol{4}$ | $\boldsymbol{-3}$ | $\boldsymbol{1}$ |
| $\boldsymbol{-4}$ | $\boldsymbol{3}$ | $\boldsymbol{-1}$ |

Since $\boldsymbol{a = 1}$, we can use the shortcut:

$$\boldsymbol{(x-4)(x+3) = 0}$$

Set each factor equal to zero:

$$\begin{array}{rcl} x - 4 &=& 0 \\ \underline{+4} && \underline{+4} \\ x &=& 4 \end{array} \qquad\qquad \begin{array}{rcl} x + 3 &=& 0 \\ \underline{-3} && \underline{-3} \\ x &=& -3 \end{array}$$

We can check to see if both values we got solve the word problem:

<u>$\boldsymbol{x = 4}$</u>

If $\boldsymbol{x = 4}$ the two consecutive intefers are $\boldsymbol{4}$ and $\boldsymbol{5}$.

- Product: $\boldsymbol{4 \cdot 5 = 20}$
- Sum: $\boldsymbol{4 + 5 = 9}$
- Is the product $\boldsymbol{11}$ more than the sum?

  $\boldsymbol{20 = 9 + 11}$ YES! $\boldsymbol{x = 4}$ is an answer!

<u>$\boldsymbol{x = -3}$</u>

If $\boldsymbol{x = -3}$, the two consecutive integers are $\boldsymbol{-3}$ and $\boldsymbol{-2}$.

- Product: $\boldsymbol{(-3)(-2) = 6}$
- Sum: $\boldsymbol{(-3) + (-2) = -5}$
- Is the product $\boldsymbol{11}$ more than the sum?

  $\boldsymbol{6 = -5 + 11}$ YES! so $\boldsymbol{x = -3}$ is also an answer!

# Applications of Quadratic Equations: Integer Problems

Practice Problems

Find two consecutive integers whose product is **1** more than their sum.