

Completing the Square

Completing the square is a useful method to solve quadratic equations that can't be factored.

Steps to solving by completing the square

- Step 1: Isolate all the variable terms on one side of the equation.
- Step 2: Calculate $\frac{1}{2}$ of the coefficient of the x term and square it.
- Step 3: Add the value obtained in step **2** to both sides of the equation.
- Step 4: Factor the new quadratic as a perfect square.
- Step 5: Use the square root property to solve the equation.

Example 1:

$$x^2 + 6x + 3 = 0$$

Since the trinomial cannot be factored, we will solve it by completing the square.

$x^2 + 6x = -3$	Step 1
$\frac{1}{2}(6) = 3 \quad 3^2 = 9$	Step 2
$x^2 + 6x + 9 = -3 + 9$	Step 3
$x^2 + 6x + 9 = 6$	Step 4
$(x + 3)^2 = 6$	
$\sqrt{(x + 3)^2} = \pm\sqrt{6}$	Step 5
$x + 3 = \pm\sqrt{6}$	Simplify
$x = -3 \pm \sqrt{6}$	Subtract 3

Example 2:

$$x^2 + 6x - \frac{13}{4} = 0$$

$$x^2 + 6x - \frac{13}{4} = 0$$

$$x^2 + 6x = \frac{13}{4}$$

$$\frac{1}{2}(6) = 3 \quad 3^2 = 9$$

$$x^2 + 6x + 9 = \frac{13}{4} + 9$$

$$x^2 + 6x + 9 = \frac{49}{4}$$

$$(x + 3)^2 = \frac{49}{4}$$

$$\sqrt{(x + 3)^2} = \pm \sqrt{\frac{49}{4}}$$

$$x + 3 = \pm \frac{7}{2}$$

$$x = -3 \pm \frac{7}{2}$$

$$x = -3 + \frac{7}{2}$$

$$x = -3 - \frac{7}{2}$$

$$x = \frac{1}{2}$$

$$x = \frac{-13}{2}$$

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Practice Problems

Solve by completing the square

1. $x^2 + 4x + 1 = 0$