The Quadratic Formula

The Quadratic Formula allows us to solve quadratic equations. It is the most useful when trying to solve quadratics that cannot be factored.

Given a quadratic equation in the form

$$\boldsymbol{ax^2 + bx + c = 0}$$

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

NOTE: The values for $\boldsymbol{a, b,}$ and $\boldsymbol{c}$ are the same values we use for the $\boldsymbol{abc}$ method.

Example 1:

Solve for $\boldsymbol{x}$ if

$$\boldsymbol{2x^2 - 5x - 3 = 0}$$

Using the quadratic formula

$$2x^2 - 5x - 3 + 0$$

$\boldsymbol{a =}$
$\boldsymbol{b =}$
$\boldsymbol{c =}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2c}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

We must follow the order of operations

$$x = \frac{5 \pm \sqrt{25 - 24}}{4}$$

$$= \frac{5 \pm \sqrt{49}}{4}$$

$$= \frac{5 \pm 7}{4}$$

We have two solutions:

$x = \frac{5+7}{4}$ and $x = \frac{5-7}{4}$

$x = \frac{12}{4}$ $x = \frac{-2}{4}$

$x = 3$ $x = -\frac{1}{2}$

Proof of the Quadratic Formula

Let $ax^2 + bx + c = 0,\ a \neq 0$

$ax^2 + bx + c = 0$	Given
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divided both sides by a
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = \left(\frac{b}{2a}\right)^2$	Add $\left(\frac{b}{2a}\right)^2$ to both sides
$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} = \left(\frac{b}{2a}\right)^2$	Complete the square (factor)
$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$	Subtract $\frac{c}{a}$ on both sides
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$	Simplify exponents
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$	Make like terms
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Combine like terms
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	Square root both sides
$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$	Simplify denominator
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	Subtract $\frac{b}{2a}$ on both sides to isolate x
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Combine like terms

The Quadratic Formula — Practice Problems

Use the quadratic formula to solve each equation.

1. $\boldsymbol{x^2 + 3x + 2 = 0}$

2. $\boldsymbol{2x^2 - 3x - 1 = 0}$

3. $\boldsymbol{x^2 + 2x = 2}$