Rationalizing the Denominator – Part 2

Rationalizing the Denominator using Conjugates

Recall: $\left(\mathbf{1}+\sqrt{\mathbf{2}}\right)\left(\mathbf{1}-\sqrt{\mathbf{2}}\right)$

$$= (1)^2 - \left(\sqrt{2}\right)^2$$
$$= 1 - 2$$
$$= -1$$

When we multiply conjugates, the result is a **rational number**.

We will use this concept to rationalize denominators.

Example 1:

Rationalize each denominator

a.) $\dfrac{\mathbf{2}}{\left(\mathbf{2}+\sqrt{\mathbf{2}}\right)}$

Multiply by the denominator's conjugate.

$$\frac{2}{\left(2+\sqrt{2}\right)}\left(\frac{2-\sqrt{2}}{2-\sqrt{2}}\right) = \frac{2\left(2-\sqrt{2}\right)}{2^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{4-2\sqrt{2}}{4-2}$$

$$= \frac{4-2\sqrt{2}}{2}$$

$$= \frac{2\left(2-\sqrt{2}\right)}{2}$$

$$= \boxed{2-\sqrt{2}}$$

b.) $\dfrac{\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

We must multiply top and bottom by $\sqrt{2}-\sqrt{3}$

$$\frac{\sqrt{3}}{\sqrt{2}+\sqrt{3}}\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}\right)=\frac{\sqrt{3}\left(\sqrt{2}-\sqrt{3}\right)}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{\sqrt{6}-\sqrt{9}}{2-3}$$

$$=\frac{\sqrt{6}-3}{-1}$$

$$=-\left(\sqrt{6}-3\right)$$

$$=\boxed{-\sqrt{6}+3 \quad \text{or} \quad 3-\sqrt{6}}$$

c.) $\dfrac{\sqrt{2}+6}{\sqrt{2}-5}$

We must multiply top and bottom by $\sqrt{2}+5$

$$\frac{\sqrt{2}+6}{\sqrt{2}-5}\left(\frac{\sqrt{2}+5}{\sqrt{2}+5}\right)=\frac{\sqrt{2}\left(\sqrt{2}+5\right)+6\left(\sqrt{2}+5\right)}{\left(\sqrt{2}\right)^2-(5)^2}$$

$$=\frac{\sqrt{4}+5\sqrt{2}+6\sqrt{2}+30}{2-25}$$

$$=\frac{2+11\sqrt{2}+30}{-23}$$

$$=\boxed{\frac{-32+11\sqrt{2}}{23}}$$

Rationalizing the Denominator Part 2

Practice Problems

Rationalize each Denominator:

1. $\frac{\mathbf{1}}{\mathbf{6-\sqrt{3}}}$

2. $\frac{\sqrt{\mathbf{5}}}{\sqrt{\mathbf{2}}+\sqrt{\mathbf{5}}}$

3. $\frac{\sqrt{\mathbf{3}}-\mathbf{1}}{\sqrt{\mathbf{3}}+\mathbf{1}}$