Solving Equations with Radicals – Part 2

Example 1:

$$\sqrt{5x+11} = x+3$$

Since the root is already isolated to one side, we can square both sides to solve:

$\sqrt{5x+11} = x+3$	Square both sides
$5x+11 = (x+3)^2$	Expand
$5x+11 = (x+3)(x+3)$	Distribute
$5x+11 = x(x+3)+3(x+3)$	Distribute
$5x+11 = x^2+3x+3x+9$	Combine like terms
$5x+11 = x^2+6x+9$	Subtract $5x$
$11 = x^2+x-9$	Subtract 11
$0 = x^2+x-2$	Factor
$0 = (x+2)(x-1)$	Set each factor equal to zero
$0 = x+2 \quad 0 = x-1$	Solve for x
$x = -2 \quad x = 1$	

Check:

When $x = -2$

$$\sqrt{5(-2)+11} = (-2)+3$$
$$\sqrt{-10+11} = 1$$
$$\sqrt{1} = 1$$
$$1 = 1$$

Therefore, True

When $x = 1$

$$\sqrt{5(1)+11} = 1+3$$
$$\sqrt{5+11} = 4$$
$$\sqrt{16} = 4$$
$$4 = 4$$

Therefore, True

So our solutions are $x = -2$ and $x = 1$

Example 2:

$$\boldsymbol{x = \sqrt{3x} + 6}$$

We want to isolate the radical term.

$\boldsymbol{x = \sqrt{3x} + 6}$	
$\boldsymbol{x - 6 = \sqrt{3x}}$	Subtract $\boldsymbol{6}$
$\boldsymbol{(x-6)^2 = 3x}$	Expand
$\boldsymbol{(x-6)(x-6) = 3x}$	Distribute
$\boldsymbol{x(x-6) - 6(x-6) = 3x}$	Distribute
$\boldsymbol{x^2 - 6x - 6x + 36 = 3x}$	Collect like terms
$\boldsymbol{x^2 - 12x + 36 = 0}$	Subtract $\boldsymbol{3x}$
$\boldsymbol{x^2 - 15x + 36 = 0}$	Factor
$\boldsymbol{(x-12)(x-3) = 0}$	Set each factor equal to $\boldsymbol{0}$
$\boldsymbol{x - 12 = 0 \quad x - 3 = 0}$	Solve for $\boldsymbol{x}$
$\boldsymbol{x = 12 \;\; x = -3}$	

Check:

When $\boldsymbol{x = 12}$

$$\boldsymbol{12 = \sqrt{3(12)} + 6}$$
$$\boldsymbol{12 = \sqrt{36} + 6}$$
$$\boldsymbol{12 = 6 + 6}$$
$$\boldsymbol{12 = 12}$$

True

When $\boldsymbol{x = -3}$

$$\boldsymbol{-3 = \sqrt{3(-3)} + 6}$$
$$\boldsymbol{-3 = \sqrt{-9} + 6}$$

$\boldsymbol{\sqrt{-9}}$ is not a real number, therefore the solution $\boldsymbol{x = -3}$ is an **extraneous** solution.

Solving Solutions with Radicals Part 2

Practice Problems

Solve:

$$x + 3 = \sqrt{x} + 9$$