Introduction to Rational Expressions

Recall: A **rational expression** is an expression that can be written in the form.

$$\frac{P}{Q}$$

Where P and Q are polynomials and Q does NOT equal zero.

If Q equals zero, we call the expression **UNDEFINED**.

Example 1:

Are there any values of x for which the following expressions is **undefined**?

a.) $\frac{x+3}{x}$

the denominator in this expression is x, therefore the expression is undefined when $x = 0$.

b.) $\frac{2x}{x-2}$

The denominator in this expression is $x - 2$, therefore the expression is undefined when

$$x - 2 = 0$$

Adding 2 to both sides we get

$$\begin{array}{r} x - 2 = 0 \\ \underline{+2 \quad +2} \\ x = 2 \end{array}$$

c.) $\dfrac{x^2-4x+1}{3}$

The denominator in this expression is 3 which is NOT zero, therefore this expression is NEVER UNDEFINED.

d.) $\dfrac{5}{x^2-4}$

The denominator in this expression is x^2-4, therefore the expression is undefined when

$$x^2-4=0$$

We need to solve for x in this equation to find out when the expression is undefined.

$x^2-4=0$		we need to factor
$(x+2)(x-2)=0$		difference to two squares
$x+2=0$	$x-2=0$	set each factor equal to 0
$x=-2$	$x=2$	solve for x.

Therefore the expression is undefined when $x=-2$ or when $x=2$.

Example 2:

Evaluate each rational expression for the given value of x.

a.) $\dfrac{x+5}{x-1}$ for $x=2$

Simply plug in 2 for x.

$$\frac{x+5}{x-1} \longrightarrow \frac{2+5}{2-1}=\frac{7}{1}=\boxed{7}$$

b.) $\frac{x^2+2x+1}{x+5}$ for $x = -1$

$$\frac{x^2+2x+1}{x+5} \rightarrow \frac{(-1)^2+2(-1)+1}{(-1)+5}$$

$$= \frac{1+(-2)+1}{4}$$

$$= \frac{-1+1}{4}$$

$$= \frac{0}{4} = \boxed{0}$$

c.) $\frac{4}{x+3}$ for $x = -3$

$$\frac{4}{x+3} \rightarrow \frac{4}{(-3)+3}$$

$$= \frac{4}{0} = \boxed{\text{UNDEFINED}}$$

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Practice Problems

For which values of $\boldsymbol{x}$ are the following expressions undefined?

1. $\dfrac{\mathbf{3}}{\boldsymbol{x}}$

2. $\dfrac{\boldsymbol{x^2+1}}{\boldsymbol{x^2-1}}$

3. $\dfrac{\boldsymbol{2x+9}}{\mathbf{4}}$

Evaluate each expression for the given value of $\boldsymbol{x}$:

4. $\dfrac{\mathbf{3}}{\boldsymbol{x}}$ for $\boldsymbol{x = 9}$

5. $\dfrac{\boldsymbol{x^2+1}}{\boldsymbol{x^2-1}}$ for $\boldsymbol{x = -1}$

6. $\dfrac{\boldsymbol{2x+9}}{\mathbf{4}}$ for $\boldsymbol{x = 0}$