Multiplying and Dividing Rational Expressions

Recall: $\frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \cdot \frac{8}{7} = \frac{3}{\cancel{4}^{1}} \cdot \frac{\cancel{8}^{2}}{7} = \frac{3 \cdot 2}{1 \cdot 7} = \boxed{\frac{6}{7}}$

Example 1:

a.) $\frac{x}{4} \div \frac{7}{8}$

$$\frac{x}{4} \div \frac{7}{8} = \frac{x}{4} \cdot \frac{8}{7} = \frac{x}{\cancel{4}^{1}} \cdot \frac{\cancel{8}^{2}}{7} = \boxed{\frac{2x}{7}}$$

b.) $\frac{x+3}{x-2} \div \frac{x-1}{x-2}$

$$\frac{x+3}{x-2} \div \frac{x-1}{x-2} = \frac{x+3}{x-2} \cdot \frac{x-2}{x-1} = \frac{x+3}{\cancel{x-2}} \cdot \frac{\cancel{x-2}}{x-1} = \boxed{\frac{x+3}{x-1}}$$

c.) $\frac{(x+2)^2}{x+1} \cdot \frac{x-1}{x+2}$

$$\frac{(x+2)^2}{x+1} \cdot \frac{x-1}{x+2}$$

Here the binomial $x + 2$ in the bottom will cancel with **one** of the $x + 2$ on the top.

If we write it out we get:

$$\frac{(x+2)^2}{x+1} \cdot \frac{x-1}{x+2} = \frac{(x+2)(x+2)}{x+1} \cdot \frac{x-1}{x+2}$$

$$= \frac{(x+2)\cancel{(x+2)}}{x+1} \cdot \frac{x-1}{\cancel{x+2}} = \boxed{\frac{(x+2)(x-1)}{(x+1)}}$$

Steps for Multiplying and Dividing Rational Expressions

Step 1: Make all division problems into multiplication problems. (See Example 1a)

Step 2: Factor the numerator and denominator completely.

Step 3: Cancel out all factors that are in the numerator AND the numerator.

> NOTE: Keep your final answer in factored form— you do not need to multiply or distribute terms.

Example 2:

$$\frac{x^2+6x+9}{x^2-4} \cdot \frac{x-2}{x+3}$$

Step 1: This step is not needed since its already a multiplication problem.

Step 2: Factor.

$$\frac{x^2+6x+9}{x^2-4} \cdot \frac{x-2}{x+3} = \frac{(x+3)(x+3)}{(x+2)(x-2)} \cdot \frac{(x-2)}{(x+3)}$$

Step 3: Cancel factors that appear on top AND bottom.

$$\frac{(x+3)\cancel{(x+3)}}{(x+2)\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}}{\cancel{(x+3)}}$$

So our final answer is $\boxed{\frac{x+3}{x+2}}$

Example 3:

$$\frac{x^2+6x+8}{x^2-x-6} \div \frac{x^2+3x-4}{x^2+2x-3}$$

Step 1: $\dfrac{x^2+6x+8}{x^2-x-6} \cdot \dfrac{x^2+2x-3}{x^2+3x-4}$

Step 2: Factor

$$\frac{x^2+6x+8}{x^2-x-6} \cdot \frac{x^2+2x-3}{x^2+3x-4} = \frac{(x+2)(x+4)}{(x-3)(x+2)} \cdot \frac{(x+3)(x-1)}{(x+4)(x-1)}$$

Step 3: Cancel factors that appear on top <u>AND</u> bottom.

$$\frac{\cancel{(x+2)}\cancel{(x+4)}}{(x-3)\cancel{(x+2)}} \cdot \frac{(x+3)\cancel{(x-1)}}{\cancel{(x+4)}\cancel{(x-1)}}$$

So our final answer is $\boxed{\dfrac{x+3}{x-3}}$

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Practice Problems

Evaluate:

1. $\frac{3x}{5} \div \frac{7x}{10}$

2. $\frac{x-2}{x+1} \div \frac{x+3}{x+1}$

3. $\frac{x^2+x-6}{x^2-3x+2} \cdot \frac{x-1}{x+4}$

4. $\frac{2x^2-x-1}{2x^2+5x+3} \cdot \frac{2x^2+x-3}{4x^2-1}$