

Applications of Rational Expressions

Work Problems

The following is an example of working together:

Example:

Joe can paint a room in **3** hours. Alice can paint the same room in **2** hours. How long will it take both Joe and Alice to paint the room if they work together?

The formula we will use for work problems is:

$$\mathbf{Rate \times Time = Task}$$

In this example, **rate** will be in terms of **rooms** per hour, **time** will be in terms of **hours**, and the **task** will be in terms of **rooms**.

$$\begin{array}{c} \mathbf{Rate \times Time = Task} \\ \downarrow \\ \frac{\mathbf{Rooms}}{\mathbf{Hour}} \times \mathbf{Hour} = \mathbf{Room} \end{array}$$

Notice that the hour units cancel.

Now we make a table to help organize the information.

	Rate	Time	Task
Joe			
Alice			

- Since rate is in terms of rooms per hour, and Joe can paint **1** room in **3** hours, Joe's rate is

$$\frac{\mathbf{1\ room}}{\mathbf{3\ hours}} \rightarrow \frac{\mathbf{1\ rooms}}{\mathbf{3\ hour}}$$

Similarly, Alice's rate is

$$\frac{\mathbf{1\ room}}{\mathbf{2\ hours}} \rightarrow \frac{\mathbf{1\ rooms}}{\mathbf{2\ hour}}$$

- Since Joe and Alice are working together, their time (in hours) will be the same. We don't know how long it will take them to complete a room together, so we call time **t**.

using the formula:

$$\mathbf{Rate \times Time = Task}$$

We can complete the table.

	Rate	Time	Task
Joe	$\frac{\mathbf{1}}{\mathbf{3}}$	\mathbf{t}	$\frac{\mathbf{t}}{\mathbf{3}}$
Alice	$\frac{\mathbf{1}}{\mathbf{2}}$	\mathbf{t}	$\frac{\mathbf{t}}{\mathbf{2}}$

The total task in this problem is to **paint one room**. In other words, when Joe and Alice work together, they will complete **1** room.

$$\text{Joe's Task} + \text{Alice's Task} = \mathbf{1\ Room}$$

$$\frac{\mathbf{t}}{\mathbf{3}} + \frac{\mathbf{t}}{\mathbf{2}} = \mathbf{1}$$

So we have the equation:

$$\frac{t}{3} + \frac{t}{2} = 1$$

Solve for t :

$$\frac{t}{3} + \frac{t}{2} = 1$$

$$\text{LCD} = 6$$

$$6\left(\frac{t}{3}\right) + 6\left(\frac{t}{2}\right) = 6(1)$$

$$2t + 3t = 6$$

$$5t = 6$$

$$t = \frac{6}{5}$$

NOTE: $\frac{6}{5}$ hours is the same as **1** hour and **12** minutes.

It makes sense that working together, Joe and Alice are able to finish the task in a shorter amount of time than it would take working alone.

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Practice Problems

Sally can clean a pool in **4** hours. It takes Bob **3** hours to clean the same pool. How long would it take Sally and Bob to clean the pool if they worked together?