Variation

Direct Variation

If y varies directly as x, then

$$y = kx$$

where k is referred to as the "constant of variation."

If x represents the number of hours you work in a week and y represents the amount of money you get paid per week we would say that y varies directly as x – as x increases, y increases and as x decreases, y also decreases.

Example 1:

Write an equation that relates x and y if y varies directly as x and when $y = 18,\ x = 3$.

Since y varies directly as x, we know

$$y = kx$$

We need to find the constant of variation, k.

When $y = 18, x = 3$

$$\begin{matrix} y = kx \\ \downarrow \quad \downarrow \\ 18 = k(3) \end{matrix}$$

Now, we solve for k.

$$\frac{18}{3} = \frac{3k}{3}$$

$$k = 6$$

So the equation that relates x then y is

$$y = 6x$$

Inverse Variation

If $\boldsymbol{y}$ varies inversely as $\boldsymbol{x}$, then

$$\boldsymbol{y = \frac{k}{x}}$$

If $\boldsymbol{x}$ represents how fast you drive to get to work and $\boldsymbol{y}$ represents how much time it takes to get to work, we would say that $\boldsymbol{y}$ varies inversely as $\boldsymbol{x}$ – as $\boldsymbol{x}$ increases, $\boldsymbol{y}$ decreases and as $\boldsymbol{x}$ decreases, $\boldsymbol{y}$ increases.

Example 2

$\boldsymbol{y}$ varies inversely as $\boldsymbol{x}$. When $\boldsymbol{x}$ is $\boldsymbol{10}$, $\boldsymbol{y}$ is $\boldsymbol{2}$. Find $\boldsymbol{x}$ when $\boldsymbol{y}$ is $\boldsymbol{4}$.

First we need to write an equation that relates $\boldsymbol{x}$ and $\boldsymbol{y}$. Since $\boldsymbol{y}$ varies inversely as $\boldsymbol{x}$, we know:

$$\boldsymbol{y = \frac{k}{x}}$$

When $\boldsymbol{x = 10}$, $\boldsymbol{y = 2}$

$$\boldsymbol{2 = \frac{k}{10}} \qquad \text{(multiply both sides by } \boldsymbol{10}\text{)}$$

$$\boldsymbol{k = 20}$$

So the equation that relates $\boldsymbol{x}$ and $\boldsymbol{y}$ is

$$\boldsymbol{y = \frac{20}{x}}$$

Now we find $\boldsymbol{x}$ when $\boldsymbol{y}$ is $\mathbf{4}$:

$$\boldsymbol{y} = \frac{\mathbf{20}}{\boldsymbol{x}}$$

$$\mathbf{4} = \frac{20}{x}$$

$$(\boldsymbol{x})4 = (\boldsymbol{x})\frac{20}{x}$$

$$\frac{\mathbf{4}\boldsymbol{x}}{\mathbf{4}} = \frac{\mathbf{20}}{\mathbf{4}}$$

$$\boldsymbol{x} = \mathbf{5}$$

Variation

Practice Problems

1. $\boldsymbol{y}$ varies directly as $\boldsymbol{x}$. When $\boldsymbol{x}$ is $\mathbf{7}$, $\boldsymbol{y}$ is $\mathbf{21}$.
   Find $\boldsymbol{y}$ when $\boldsymbol{x}$ is $\mathbf{5}$.

2. $\boldsymbol{y}$ varies inversely as $\boldsymbol{x}$. When $\boldsymbol{x}$ is $\mathbf{2}$, $\boldsymbol{y}$ is $\mathbf{14}$.
   Find $\boldsymbol{y}$ when $\boldsymbol{x}$ is $\mathbf{28}$.