Math251

Practice Exam #03

1. Find each square.

a) $(a-b)^2$ b) $\left(4a-\frac{5}{4}b\right)^2$ c) $-a(a-b)^2$

2. Find each product.

a) $(9x-2y)(9x+2y)$ b) $(x-3)(x+3)$ c) $a(ab-c)(ab+c)$

3. Simplify each expression. Write answers with only positive exponents.

a) $\dfrac{\left(x^{-5}\right)^{-4}}{x^{-2}}$ b) $\dfrac{\left(b^2+a^2\right)^{-4}}{\left(b^2+a^2\right)^{-7}}$ c) $\dfrac{\left(x^3y^{-4}z^2\right)^{-1}}{\left(x^{-1}y^{-2}z^{-1}\right)^{-2}}$

4. Simplify.

a) $\left(4x^2-3x+2\right)-\left(-9x^2-5x+1\right)$ b) $a^2b-4a-ab^4+4b^3$

5. Factor completely.

a) $2x^2+15x+7$ b) $6x^2-13x-8$ c) $16x^2+65x+4$

6. Factor Completely.

a) $14x^3y^4 + 3x^2y^5 - 2xy^6$

b) $2y^3z - 3y^2z^2 - 27yz^3$

7. Solve for x.

a) $x^2 - 3ax - 10a^2 = 0$

b) $3x(x-5) = -12$

8. Factor Completely. Use the note below for parts b) and c).

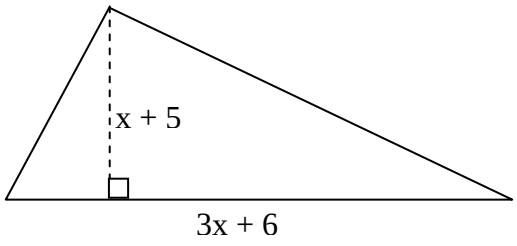
Note: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

a) $x^4 - 81$

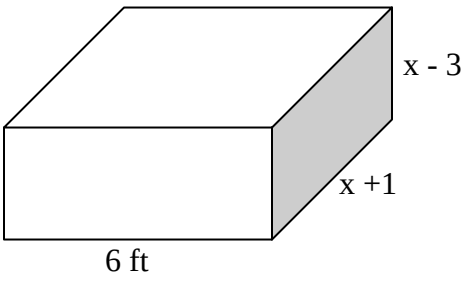
b) $y^3 + 8$

c) $27y^3 - 8$

9. The area of this triangle is 60 square inches. Find the height and the base of this triangle. Set up an equation an solve. $A = \frac{1}{2}bh$

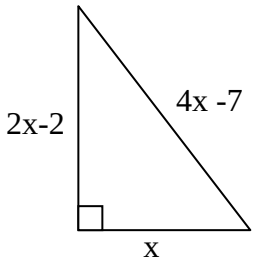


10. The box below has a volume of 72 ft^3. Set up an equation and solve for x. Then label the width and the height of the box.



11. Write out an equation that relates the three sides of the right triangle below. Then solve this equation for x. Then label the length of the sides of the triangle.

$c^2 = a^2 + b^2$



Math251

Practice Exam #03

1. Find each square.

a) $(a-b)^2$

$=(a-b)(a-b)$

$=a(a-b)-b(a-b)$

$=a^2-ab-ab+b^2$

$=\boxed{a^2-2ab+b^2}$

b) $\left(4a-\frac{5}{4}b\right)^2$

$=\left(4a-\frac{5}{4}b\right)\left(4a-\frac{5}{4}b\right)$

$=4a\left(4a-\frac{5}{4}b\right)-\frac{5}{4}b\left(4a-\frac{5}{4}b\right)$

$=16a^2-5ab-5ab+\frac{25}{16}b^2$

$=\boxed{16a^2-10ab+\frac{25}{16}b^2}$

c) $-a(a-b)^2$

$=-a(a-b)(a-b)$

$=-a[a(a-b)-b(a-b)]$

$=-a[a^2-ab-ab+b^2]$

$=-a[a^2-2ab+b^2]$

$=\boxed{-a^3+2a^2b-ab^2}$

2. Find each product.

a) $(9x-2y)(9x+2y)$

$=9x(9x-2y)+2y(9x-2y)$

$=81x^2-18xy+18xy-4y^2$

$=\boxed{81x^2-4y^2}$

b) $(x-3)(x+3)$

$=x(x-3)+3(x-3)$

$=x^2-3x+3x-9$

$=\boxed{x^2-9}$

c) $a(ab-c)(ab+c)$

$=a[ab(ab-c)+c(ab-c)]$

$=a[a^2b^2-abc+abc-c^2]$

$=a[a^2b^2-c^2]$

$=\boxed{a^3b^2-ac^2}$

3. Simplify each expression. Write answers with only positive exponents.

a) $\dfrac{\left(x^{-5}\right)^{-4}}{x^{-2}}$

$= \dfrac{x^{20}}{x^{-2}} = x^{20} \cdot x^{2}$

$= \boxed{x^{22}}$

b) $\dfrac{\left(b^2+a^2\right)^{-4}}{\left(b^2+a^2\right)^{-7}}$

$= \dfrac{\left(b^2+a^2\right)^{7}}{\left(b^2+a^2\right)^{4}}$

$= \boxed{\left(b^2+a^2\right)^{3}}$

c) $\dfrac{\left(x^3y^{-4}z^2\right)^{-1}}{\left(x^{-1}y^{-2}z^{-1}\right)^{-2}}$

$= \dfrac{x^{-3}y^{4}z^{-2}}{x^{2}y^{4}z^{2}}$

$= \dfrac{y^4}{x^2 x^3 \cdot y^4 \cdot z^2 z^2} = \dfrac{y^4}{x^5 y^4 \cdot z^4}$

$= \boxed{\dfrac{1}{x^5 z^4}}$

4. Simplify.

a) $\left(4x^2-3x+2\right)-\left(-9x^2-5x+1\right)$

$= 4x^2-3x+2+9x^2+5x-1$

$= \boxed{13x^2+2x+1}$

b) $a^2b-4a-ab^4+4b^3$

$= a(ab-4)-b^3(ab-4)$

$= \boxed{(ab-4)(a-b^3)}$

5. Factor completely.

a) $2x^2+15x+7$

$a=2$, $b=15$, $c=7$

$a \cdot c = 14$

	sum
14, 1	15

$= 2x^2+14x+x+7$

$= 2x(x+7)+1(x+7)$

$= \boxed{(x+7)(2x+1)}$

b) $6x^2-13x-8$

$a=6$, $b=-13$, $c=-8$

$a \cdot c = -48$

	sum
$-48 \cdot 1$	-47
$-24 \cdot 2$	-22
$-12 \cdot 4$	-8
$-6 \cdot 8$	2
$-3 \cdot 16$	13
$3 \cdot -16$	-13

$= 6x^2+3x-16x-8$

$= 3x(2x+1)-8(2x+1)$

$= \boxed{(2x+1)(3x-8)}$

c) $16x^2+65x+4$

$a=16$, $b=65$, $c=4$

$a \cdot c = 64$

	sum
64, 1	65

$= 16x^2+64x+x+4$

$= 16x(x+4)+1(x+4)$

$= \boxed{(x+4)(16x+1)}$

6. Factor Completely.

a) $14x^3y^4 + 3x^2y^5 - 2xy^6$

$= xy^4(14x^2 + 3xy - 2y^2)$

$a = 14$	$a \cdot c = -28$	sum
$b = 3$	$-28 \cdot 1$	-27
$c = -2$	$-14 \cdot 2$	-12
	$-7 \cdot 4$	-3
	$7 \cdot -4$	3

$= xy^4[14x^2 + 7xy - 4xy - 2y^2]$

$= xy^4[7x(2x+y) - 2y(2x+y)]$

$= \boxed{xy^4(2x+y)(7x-2y)}$

b) $2y^3z - 3y^2z^2 - 27yz^3$

$= yz[2y^2 - 3yz - 27z^2]$

$a = 2$	$a \cdot c = -54$	sum
$b = -3$	$-54 \cdot 1$	-53
$c = -27$	$-27 \cdot 2$	-25
	$-9 \cdot 6$	-3

$= yz[2y^2 - 9yz + 6yz - 27z^2]$

$= yz[y(2y-9z) + 3z(2y-9z)]$

$= \boxed{yz(2y-9z)(y+3z)}$

7. Solve for x.

a) $x^2 - 3ax - 10a^2 = 0$

$a = 1$	$a \cdot c = -10$	sum
$b = -3$	$-10 \cdot 1$	-9
$c = -10$	$-5 \cdot 2$	-3

$x^2 - 5ax + 2ax - 10a^2 = 0$

$x(x-5a) + 2a(x-5a) = 0$

$(x-5a)(x+2a) = 0$

$x - 5a = 0$ (+5a +5a) $\quad x + 2a = 0$ (−2a −2a)

$\boxed{x = 5a} \qquad \boxed{x = -2a}$

b) $3x(x-5) = -12$

$3x^2 - 15x = -12$

$\frac{1}{3}[3x^2 - 15x] = \frac{1}{3}[-12]$

$x^2 - 5x = -4$

$+4 \quad +4$

$x^2 - 5x + 4 = 0$

$a = 1$	$a \cdot c = 4$	sum
$b = -5$	$-4 \cdot -1$	-5
$c = 4$		

$a = 1$: shortcut! $(x-4)(x-1) = 0$

$x - 4 = 0$ $\quad x - 1 = 0$

$\boxed{x = 4} \qquad \boxed{x = 1}$

8. Factor Completely. Use the note below for parts b) and c).

Note: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

a) $x^4 - 81$

$= (x^2)^2 - 9^2$

$= (x^2+9)(x^2-9)$

$= \boxed{(x^2+9)(x+3)(x-3)}$

b) $y^3 + 8$

$= y^3 + 2^3$

$= (y+2)(y^2 - 2 \cdot y + 2^2)$

$= \boxed{(y+2)(y^2 - 2y + 4)}$

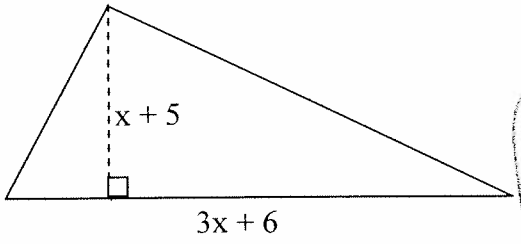
c) $27y^3 - 8$

$= (3y)^3 - 2^3$

$= (3y-2)[(3y)^2 + (3y)(2) + 2^2]$

$= \boxed{(3y-2)(9y^2 + 6y + 4)}$

9. The area of this triangle is 60 square inches. Find the height and the base of this triangle. Set up an equation an solve. $A = \frac{1}{2}bh$



$60 = \frac{1}{2}(3x+6)(x+5)$

$60 = \frac{1}{2}[x(3x+6)+5(3x+6)]$

$60 = \frac{1}{2}[3x^2+6x+15x+30]$

$60 = \frac{1}{2}[3x^2+21x+30]$

★ mult both sides by 2: $120 = 3x^2+21x+30$

★ divide both sides by 3: $40 = x^2+7x+10$

$-40 \quad -40$

$0 = x^2+7x-30$

$x^2+7x-30=0$

$a=1$, $b=7$, $c=-30$

$a \cdot c = -30$

	sum
$-30 \cdot 1$	-29
$-15 \cdot 2$	-13
$-5 \cdot 6$	1
$-10 \cdot 3$	-7
$10 \cdot -3$	7

$a=1$: short cut!

$(x+10)(x-3)=0$

$x+10=0 \quad x-3=0$

$-10 \quad -10 \quad +3 \quad +3$

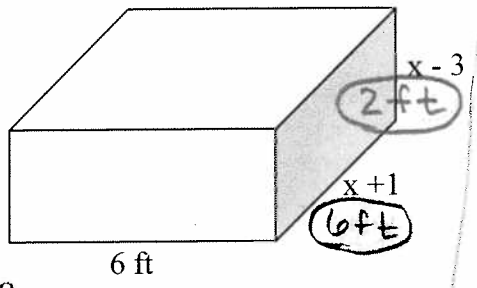
$x=-10 \quad x=3$

$x=3$: $b = 3(3)+6 = 9+6 = 15$

$h = (3)+5 = 8$

$b=15,\ h=8$

10. The box below has a volume of 72 ft^3. Set up an equation and solve for x. Then label the width and the height of the box.



$V = lwh$

$72 = 6(x+1)(x-3)$

★ divide both sides by 6: $12 = (x+1)(x-3)$

$12 = x(x+1)-3(x+1)$

$12 = x^2+x-3x-3$

$12 = x^2-2x-3$

$-12 \quad -12$

$0 = x^2-2x-15$

$x^2-2x-15=0$

$a=1$, $b=-2$, $c=-15$

$a \cdot c = -15$

	sum
$-15 \cdot 1$	-14
$-5 \cdot 3$	-2

$a=1$: Short cut!

$(x-5)(x+3)=0$

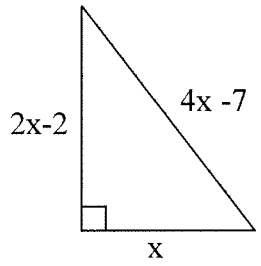
$x-5=0 \quad x+3=0$

$+5 \quad +5 \quad -3 \quad -3$

$x=5 \quad x=-3$

11. Write out an equation that relates the three sides of the right triangle below. Then solve this equation for x. Then label the length of the sides of the triangle.

$c^2 = a^2 + b^2$



$(4x-7)^2 = (2x-2)^2 + x^2$

$(4x-7)(4x-7) = (2x-2)(2x-2) + x^2$

$4x(4x-7)-7(4x-7) = 2x(2x-2)-2(2x-2) + x^2$

$16x^2-28x-28x+49 = 4x^2-4x-4x+4+x^2$

$16x^2-56x+49 = 5x^2-8x+4$

$-5x^2+8x-4 \quad -5x^2+8x-4$

$11x^2-48x+45=0$

$a=11$
$b=-48$
$c=45$

$a \cdot c = 495$

	SUM
495 1	496
165 3	168
55 9	64
11 45	56
33 15	48
−33 −15	−48

$11x^2-33x-15x+45=0$

$11x(x-3)-15(x-3)=0$

$(x-3)(11x-15)=0$

$x-3=0$ $\quad 11x-15=0$

$+3 \quad +3 \quad +15 \quad +15$

$x=3 \quad \frac{11x}{11}=\frac{15}{11}$

$x=\frac{15}{11}$

If $x=\frac{15}{11}$, then

$4x-7 = 4\left(\frac{15}{11}\right)-7$

$= \frac{60}{11}-7$

$= \frac{60}{11}-\frac{77}{11}$

$= -\frac{17}{11}$

Not Possible!

SOLUTION

So, $x=3$.