

# Math251

## Practice Exam #04

SOLUTIONS

1. Multiply or divide as indicated.

a)  $\frac{8x-24}{3x+12} \cdot \frac{7x+28}{5x-15}$

b)  $\frac{x^2-3x-10}{x^2-4x-5} \div \frac{4x+8}{8x+8}$

2. Add or Subtract as indicated and simplify.

a)  $\frac{-2p}{p-4} - \frac{8}{4-p}$

b)  $\frac{1}{2x^2+13x+6} - \frac{1}{2x^2-x-1}$

3. Simplify each complex fraction.

$$\text{a) } \frac{\frac{2}{x} - 2}{5 - \frac{5}{x}}$$

$$\text{b) } \frac{\frac{1}{x^3y} + \frac{2}{xy^2}}{\frac{4}{xy} + \frac{1}{x^2y}}$$

4. Solve each equation.

$$\text{a) } \frac{3x}{4} - \frac{x-5}{3} = x$$

$$\text{b) } \frac{2x}{x-3} + \frac{1}{x+3} = \frac{2x}{x^2-9}$$

5. A large pump can empty a pool in 6 hours, and a smaller pump can do it in 24 hours. How long will it take the two pumps working together to empty half of the pool?

**Complete the table and set up an equation to solve this problem**

	Rate	$\times$	Time	$=$	Task
Large					
Small					

6. If  $x$  varies inversely as  $y$ , and  $x = \frac{1}{10}$  when  $y = 5$ , find  $y$  when  $x = 6$ .

7. Sally can row 5 mph in still water. It takes her the same amount of time to row 30 miles upstream as it does to row 90 miles downstream. How fast is the current?

**Complete the table and set up an equation to solve this problem.**

	Rate	Time	Distance
upstream			
downstream			

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## Practice Exam #04

SOLUTIONS

1. Multiply or divide as indicated.

$$\begin{aligned} \text{a) } & \frac{8x-24}{3x+12} \cdot \frac{7x+28}{5x-15} \\ & = \frac{8(x-3)}{3(x+4)} \cdot \frac{7(x+4)}{5(x-3)} \\ & = \frac{8(x-3) \cdot 7(x+4)}{3(x+4) \cdot 5(x-3)} \\ & = \boxed{\frac{56}{15}} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{x^2-3x-10}{x^2-4x-5} \div \frac{4x+8}{8x+8} \\ & = \frac{x^2-3x-10}{x^2-4x-5} \cdot \frac{8x+8}{4x+8} \\ & = \frac{(x-5)(x+2)}{(x-5)(x+1)} \cdot \frac{8(x+1)}{4(x+2)} \\ & = \frac{(x-5)(x+2) \cdot 8(x+1)}{(x-5)(x+1) \cdot 4(x+2)} = \frac{2}{1} = \boxed{2} \end{aligned}$$

Note: For both trinomials,  $a=1$ , so we can use the shortcut!

2. Add or Subtract as indicated and simplify.

$$\begin{aligned} \text{a) } & \frac{-2p}{p-4} - \frac{8}{4-p} \\ & = \frac{-2p}{p-4} - \frac{8}{-(p-4)} \\ & = \frac{-2p}{p-4} + \frac{8}{p-4} \\ & = \frac{-2p+8}{p-4} = \frac{-2(p-4)}{p-4} \\ & = \boxed{-2} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{1}{2x^2+13x+6} - \frac{1}{2x^2-x-1} \\ & \quad \swarrow \quad \searrow \\ & \quad \begin{array}{l} a=2 \\ b=13 \\ c=6 \\ a \cdot c = 12 \\ \begin{array}{c} \wedge \\ \text{12} \quad \text{1} \\ \text{Sum} \end{array} \end{array} \quad \begin{array}{l} a=2 \\ b=-1 \\ c=-1 \\ a \cdot c = -2 \\ \begin{array}{c} \wedge \\ -2 \quad 1 \\ \text{Sum} \end{array} \end{array} \\ & \quad \frac{2x^2+12x+x+6}{2x(x+6)+1(x+6)} \quad \frac{2x^2-2x+x-1}{2x(x-1)+1(x-1)} \\ & \quad = \frac{2x(x+6)+1(x+6)}{(2x+1)(x+6)} \quad = \frac{(2x+1)(x-1)}{(2x+1)(x-1)} \\ & = \frac{1}{(2x+1)(x+6)} - \frac{1}{(2x+1)(x-1)} \\ & \quad \text{LCD} = (2x+1)(x+6)(x-1) \\ & = \frac{(x-1) - (x+6)}{(2x+1)(x+6)(x-1)} \\ & = \frac{x-1-x-6}{(2x+1)(x+6)(x-1)} = \boxed{\frac{-7}{(2x+1)(x+6)(x-1)}} \end{aligned}$$

3. Simplify each complex fraction.

$$\text{a) } \frac{\frac{2}{x} - 2}{5 - \frac{5}{x}} \quad \text{LCD} = x$$

$$\begin{aligned} \frac{x \left( \frac{2}{x} - 2 \right)}{x \left( 5 - \frac{5}{x} \right)} &= \frac{2 - 2x}{5x - 5} \\ &= \frac{2(1-x)}{5(x-1)} = \frac{-2(x-1)}{5(x-1)} \\ &= \boxed{-\frac{2}{5}} \end{aligned}$$

$$\text{b) } \frac{\frac{1}{x^3y} + \frac{2}{xy^2}}{\frac{4}{xy} + \frac{1}{x^2y}} \quad \text{LCD} = x^3y^2$$

$$\begin{aligned} &= \frac{x^3y^2 \left[ \frac{1}{x^3y} + \frac{2}{xy^2} \right]}{x^3y^2 \left[ \frac{4}{xy} + \frac{1}{x^2y} \right]} \\ &= \frac{y + 2x^2}{4x^2y + xy} = \boxed{\frac{y + 2x^2}{xy(4x+1)}} \end{aligned}$$

4. Solve each equation.

$$\text{a) } \frac{3x}{4} - \frac{x-5}{3} = x \quad \text{LCD} = 12$$

$$12 \left[ \frac{3x}{4} - \frac{(x-5)}{3} \right] = 12[x]$$

$$3(3x) - 4(x-5) = 12x$$

$$9x - 4(x-5) = 12x$$

$$9x - 4x + 20 = 12x$$

$$5x + 20 = 12x$$

$$\begin{array}{r} -5x \qquad \qquad -5x \\ \hline 20 = 7x \end{array}$$

$$\frac{20}{7} = \frac{7x}{7}$$

$$\boxed{x = \frac{20}{7}}$$

$$\text{b) } \frac{2x}{x-3} + \frac{1}{x+3} = \frac{2x}{x^2-9}$$

$$\frac{2x}{x-3} + \frac{1}{x+3} = \frac{2x}{(x+3)(x-3)}$$

$$\text{LCD} = (x+3)(x-3)$$

$$(x+3)(x-3) \left[ \frac{2x}{x-3} + \frac{1}{x+3} \right] = (x+3)(x-3) \left[ \frac{2x}{(x+3)(x-3)} \right]$$

$$2x(x+3) + (x-3) = 2x$$

$$2x^2 + 6x + x - 3 = 2x$$

$$2x^2 + 7x - 3 = 2x$$

$$\begin{array}{r} -2x \qquad \qquad -2x \\ \hline 2x^2 + 5x - 3 = 0 \end{array}$$

$$2x^2 + 5x - 3 = 0$$

a=2	a.c = -6	SUM
b=5	^	
c=-3	-6 1	-5
	6 -1	5

$$2x^2 + 6x - x - 3 = 0$$

$$2x(x+3) - 1(x+3) = 0$$

$$(x+3)(2x-1) = 0$$

$$x+3=0 \quad 2x-1=0$$

$$\boxed{x = -3} \quad 2x = 1 \Rightarrow \boxed{x = \frac{1}{2}}$$

Note: This is an **EXTRANEIOUS** SOLUTION

5. A large pump can empty a pool in 6 hours, and a smaller pump can do it in 24 hours. How long will it take the two pumps working together to empty half of the pool?

**Complete the table and set up an equation to solve this problem**

	Rate	Time	Task
Large	$\frac{1}{6}$	$t$	$\frac{t}{6}$
Small	$\frac{1}{24}$	$t$	$\frac{t}{24}$
			$\frac{1}{2}$

$$\frac{t}{6} + \frac{t}{24} = \frac{1}{2}$$

$$24 \left[ \frac{t}{6} + \frac{t}{24} \right] = 24 \left[ \frac{1}{2} \right]$$

$$4t + t = 12$$

$$5t = 12$$

$$t = \frac{12}{5} \text{ hrs.}$$

6. If  $x$  varies inversely as  $y$ , and  $x = \frac{1}{10}$  when  $y = 5$ , find  $y$  when  $x = 6$ .

$$x = \frac{k}{y}$$

$$\frac{1}{10} = \frac{k}{5}$$

$$5 = 10k$$

$$k = \frac{5}{10}$$

$$k = \frac{1}{2}$$

$$x = \frac{\frac{1}{2}}{y}$$

$$6 = \frac{\frac{1}{2}}{y}$$

$$6y = \frac{1}{2}$$

$$y = \frac{1}{12}$$

7. Sally can row 5 mph in still water. It takes her the same amount of time to row 30 miles upstream as it does to row 90 miles downstream. How fast is the current?

**Complete the table and set up an equation to solve this problem.**

	Rate	Time	Distance
upstream	$5-x$	$\frac{30}{5-x}$	30
downstream	$5+x$	$\frac{90}{5+x}$	90

$$R_{\text{Boat}} = 5 \text{ mph}$$

$$R_{\text{Current}} = x$$

$$\frac{30}{5-x} = \frac{90}{5+x}$$

$$\text{LCD} = (5-x)(5+x)$$

$$(5-x)(5+x) \left[ \frac{30}{5-x} \right] = (5-x)(5+x) \left[ \frac{90}{5+x} \right]$$

$$30[5+x] = 90[5-x]$$

$$150 + 30x = 450 - 90x$$

$$+90x \qquad +90x$$

$$150 + 120x = 450$$

$$-150$$

$$-150$$

$$120x = 300$$

$$\frac{120x}{120} = \frac{300}{120}$$

$$x = \frac{5}{2} \text{ mph}$$