Solving Systems of Equations:
Substitution

Example 1:

$$x - y = 2$$
$$x + y = 6$$

Method of Substitution:

i.) Take one equation and solve for $\boldsymbol{x}$ or $\boldsymbol{y}$ (it doesn't matter which order)
ii.) Substitute into the other equation.
iii.) Solve for each variable.

i.) Using the first equation, let's solve for $\boldsymbol{x}$.

$$\begin{array}{r} x - y = 2 \\ \underline{\quad +y \quad +y} \\ x = 2 + y \end{array}$$

ii.) Now we substitute this equation into the second equation.

$$x + y = 6$$
$$(2 + y) + y = 6$$

iii.) And now we can solve for $\boldsymbol{y}$.

$$\begin{array}{r} 2 + y + y = 6 \\ 2 + 2y = 6 \\ \underline{-2 \qquad\quad -2} \\ 2y = 4 \end{array}$$
$$\frac{2y}{2} = \frac{4}{2}$$

$$\boxed{y = 2}$$

Now that we know that $\boldsymbol{y = 2}$, we can easily solve for $\boldsymbol{x}$ since from part i.) we found that:

$$\boldsymbol{x = 2 + y}$$
$$\boldsymbol{x = 2 + 2}$$
$$\boxed{\boldsymbol{x = 4}}$$

> **<u>NOTE:</u>** We could have plugged $\boldsymbol{y = 2}$ in either original equation and still obtained $\boldsymbol{x = 4}$.

So, what does this mean?

Since $\boldsymbol{x = 4}$ and $\boldsymbol{y = 2}$, this means that the point $(4,2)$ satisfies both equations.

In other words, these two lines intersect at the point $\boldsymbol{(4, 2)}$.

Example 2:

$$\boldsymbol{4x + 7y = 13}$$
$$\boldsymbol{x + y = 1}$$

Using the substitution method, we want to choose one equation and solve for either $\boldsymbol{x}$ or $\boldsymbol{y}$. Since the second equation is a bit less complicated than the first, we'll choose to use the second.

i.) Using the second equation, let's solve for $\boldsymbol{y}$ -remember, it doesn't matter which variable you choose to solve for!

$$\boldsymbol{x + y = 1}$$
$$\underline{\boldsymbol{-x \qquad - x}}$$
$$\boxed{\boldsymbol{y = 1 - x}}$$

ii.) Now we substitute this equation into the FIRST equation.

$$4x + 7y = 13$$
$$4x + 7(1 - x) = 13$$

iii.) And now we can solve for x.

$$4x + 7(1 - x) = 13$$
$$4x + 7 - 7x = 13$$

$$\begin{aligned} -3x + 7 &= 13 \\ -7 \quad &\;\; -7 \\ \hline -3x \quad &= \;\; 6 \end{aligned}$$

$$\frac{-3x}{-3} = \frac{6}{-3}$$

$$\boxed{x = -2}$$

Now that we know that $x = -2$, we can easily solve for y since from part i.) we found that

$$y = 1 - x$$

$$y = 1 - (-2)$$

$$y = 1 + 2$$

$$\boxed{y = 3}$$

This means that the two lines intersect at the point (_____ , ______).

Solving Systems: Substitution Practice Problems

Solve each system of equations:

1. $x + y = 2$
   $x - y = -4$

2. $x + y = 5$
   $2x - 3y = 0$