

§3-2**QUADRATIC EQUATIONS****Definition**

Quadratic equations in one variable are equations which can be written in the form $ax^2 + bx + c = 0$ where a , b and c are real numbers and $a \neq 0$.

Quadratic equations can have two real solutions, one real solution or no real solutions. They can often be solved by factoring and applying the zero-product property.

Property**The Zero-product Property**

For any two real numbers a and b if $ab = 0$ then $a = 0$ or $b = 0$.

Example 1

Solve the equation for x , $x^2 = x + 2$.

Solution

$$\begin{aligned}x^2 &= x + 2 \\x^2 - x - 2 &= 0 \\(x - 2)(x + 1) &= 0 \\x - 2 = 0 &\quad \text{or} \quad x + 1 = 0 \\x = 2 &\quad \quad \quad x = -1\end{aligned}$$

Example 2

Solve the equation for x , $3x^2 - 6x + 4 = -x^2 + 6x - 5$.

Solution

$$\begin{aligned}3x^2 - 6x + 4 &= -x^2 + 6x - 5 \\4x^2 - 12x + 9 &= 0 \\(2x - 3)^2 &= 0 \\\sqrt{(2x - 3)^2} &= \pm\sqrt{0} \\2x - 3 &= 0 \\2x &= 3 \\x &= \frac{3}{2}\end{aligned}$$

Example 3

Solve the equation for x , $2x^2 + 18 = 0$.

Solution

$$\begin{aligned}2x^2 + 18 &= 0 \\2x^2 &= -18 \\x^2 &= -9 \\\sqrt{x^2} &= \pm\sqrt{-9} \\x &= \pm 3i\end{aligned}$$

Thus there are no real solutions.

For some quadratic equations it is necessary to use the *quadratic formula*.

Formula

The Quadratic Formula

$$\text{For any quadratic equation } ax^2 + bx + c = 0, x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 4 Solve the quadratic equation for x : $2x^2 + 3x - 5 = 0$.

Solution
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-5)}}{2(2)} = \frac{-3 \pm \sqrt{49}}{4} = \frac{-3 \pm 7}{4}$$

$$\text{So } x = \frac{-3+7}{4} = \frac{4}{4} = 1 \text{ or } x = \frac{-3-7}{4} = \frac{-10}{4} = -\frac{5}{2}.$$

Example 5 Solve the quadratic equation for x : $x^2 + 3x - 5 = 0$.

Solution
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)} = \frac{-3 \pm \sqrt{29}}{4}$$

$$\text{So } x = \frac{-3+\sqrt{29}}{2} \approx 1.19 \text{ or } x = \frac{-3-\sqrt{29}}{2} \approx 4.19.$$

Example 6 Solve the quadratic equation for x : $x^2 - x + 1 = 0$.

Solution
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(1)}}{2(2)} = \frac{2 \pm \sqrt{-4}}{4} = \frac{2 \pm 2i}{4} = \frac{1 \pm i}{2}$$

$$\text{So } x = \frac{1+i}{2} \text{ or } x = \frac{1-i}{2}.$$

Example 7 The sum of three times x and twice the square of x is equal to seven. Solve the quadratic equation for x .

Solution $2x^2 + 3x = 7$ so $2x^2 + 3x - 7 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-7)}}{2(2)} = \frac{-3 \pm \sqrt{51}}{4}$$

$$\text{So } x = \frac{-3+\sqrt{51}}{4} \text{ or } x = \frac{-3-\sqrt{51}}{4}.$$

Solve each quadratic equation by factoring.

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|-------------------------|---|-----------------------------|
| 1. $y^2 - 17y + 70 = 0$ | 2. $x^2 + 9x + 13 = -7$ | 3. $x(x+1) = 112 - 5x$ |
| 4. $a^2 + 25 = 10a$ | 5. $2d^2 + 5d = 12$ | 6. $a^2 + 3a + 2 = -3(a+2)$ |
| 7. $10 - 9y = -2y^2$ | 8. $2x^2 = 5x - 2$ | 9. $c(c+4) = 3 + 3(9+c)$ |
| 10. $a^2 = 4(2a-3)$ | 11. $b(b+3) = -2$ | 12. $2a(a+6) = 5 - a(a+2)$ |
| 13. $(x+3)^2 = 6(x+15)$ | 14. $10(10+y^2) - 9(y^2+2) - 2(1+y^2) = -1$ | |

Solve each quadratic equation using the quadratic formula.

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|--------------------------|---|-------------------------|
| 15. $2x^2 + 7x - 15 = 0$ | 16. $3x^2 - x - 14 = 0$ | 17. $t^2 + 4t + 2 = 0$ |
| 18. $w^2 + 2w + 4 = 0$ | 19. $2w^2 + w - 2 = 0$ | 20. $3k^2 + 5k + 1 = 0$ |
| 21. $x^2 - 4x + 4 = 0$ | 22. $9x^2 + 6x + 1 = 0$ | 23. $4x^2 - 12x = 0$ |
| 24. $x^2 + 4 = 0$ | 25. $-x^2 = 5x + 20$ | 26. $(2x+1)(x+1) = 7$ |
| 27. $x^2 = 1 + x$ | 28. $\frac{1}{2}x^2 + \frac{1}{6}x - 1 = 0$ | 29. $0.5t^2 = t + 0.5$ |

Solve each quadratic equation using any method.

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| 30. $9x^2 + 12x + 4 = 0$ | 31. $9x^2 + 1 = 0$ | 32. $x^2 + 5x + 1 = 0$ |
| 33. $3x^2 - 16x + 5 = 0$ | 34. $x^2 = 5(x-1)$ | 35. $x - \frac{1}{x} = 4 - \frac{x}{2} + \frac{1}{2x}$ |
| 36. $\frac{x^2}{4} + 2 = \frac{3x}{2}$ | 37. $x^2 - 14x + 45 = 0$ | 38. $-3x^2 + 5x + 12 = 0$ |
| 39. $9(x-1)^2 - 1 = 0$ | 40. $1.9x = 4.3x^2 - 9$ | 41. $\frac{x-4}{2} + \frac{x^2+x}{3} = 0$ |
| 42. $3x^2 + 7x - 26 = 0$ | 43. $x^2 + \frac{7}{6}x = \frac{1}{2}$ | 44. $(x-3)(x+2) = 14$ |
| 45. $x = 3 + \frac{4x-12}{x}$ | 46. $5x^2 + 15x = 0$ | 47. $(x-2)(x+4) + 16 = 4x^2$ |
| 48. $4x^2 - 49 = 0$ | 49. $2x^2 + 5x - 3 = 0$ | 50. $\frac{x+3}{x} - \frac{x}{x+2} = 8$ |

1. $\{7, 10\}$ 2. $\{-5, -4\}$ 3. $\{-14, 8\}$ 4. $\{5\}$
5. $\left\{-4, \frac{3}{2}\right\}$ 6. $\{-4, -2\}$ 7. $\left\{2, \frac{5}{2}\right\}$ 8. $\left\{\frac{1}{2}, 2\right\}$
9. $\{-6, 5\}$ 10. $\{2, 6\}$ 11. $\{-2, -1\}$ 12. $\left\{-5, \frac{1}{3}\right\}$
13. $\{-9, 9\}$ 14. $\{-9, 9\}$ 15. $\left\{-5, \frac{3}{2}\right\}$ 16. $\left\{-2, \frac{7}{3}\right\}$
17. $\{-2 \pm \sqrt{2}\}$ 18. $\{-1 \pm i\sqrt{3}\}$ 19. $\left\{\frac{-1 \pm \sqrt{17}}{4}\right\}$ 20. $\left\{\frac{-5 \pm \sqrt{13}}{6}\right\}$
21. $\{2\}$ 22. $\left\{-\frac{1}{3}\right\}$ 23. $\{0, 3\}$ 24. $\{\pm 2i\}$
25. $\left\{\frac{-5 \pm i\sqrt{55}}{2}\right\}$ 26. $\left\{\frac{-3 \pm \sqrt{57}}{4}\right\}$ 27. $\left\{\frac{1 \pm \sqrt{5}}{2}\right\}$
28. $\left\{\frac{-1 \pm \sqrt{73}}{6}\right\}$ 29. $\{1 \pm \sqrt{2}\}$ 30. $\left\{-\frac{2}{3}\right\}$
31. $\left\{\pm \frac{i}{3}\right\}$ 32. $\left\{\frac{-6 \pm \sqrt{21}}{2}\right\}$ 33. $\left\{\frac{1}{3}, 5\right\}$
34. $\left\{\frac{5 \pm \sqrt{5}}{2}\right\}$ 35. $\left\{-\frac{1}{3}, 3\right\}$ 36. $\{2, 4\}$
37. $\{5, 9\}$ 38. $\left\{-\frac{4}{3}, 3\right\}$ 39. $\left\{\frac{2}{3}, \frac{4}{3}\right\}$
40. $\left\{\frac{19 \pm \sqrt{15,841}}{86}\right\}$ 41. $\left\{-4, \frac{3}{2}\right\}$ 42. $\left\{2, -\frac{13}{3}\right\}$
43. $\left\{\frac{1}{3}, -\frac{3}{2}\right\}$ 44. $\{-4, 5\}$ 45. $\{3, 4\}$
46. $\{-3, 0\}$ 47. $\left\{-\frac{4}{3}, 2\right\}$ 48. $\left\{\pm \frac{7}{2}\right\}$
49. $\left\{-3, \frac{1}{2}\right\}$ 50. $\left\{\frac{-11 \pm \sqrt{313}}{16}\right\}$