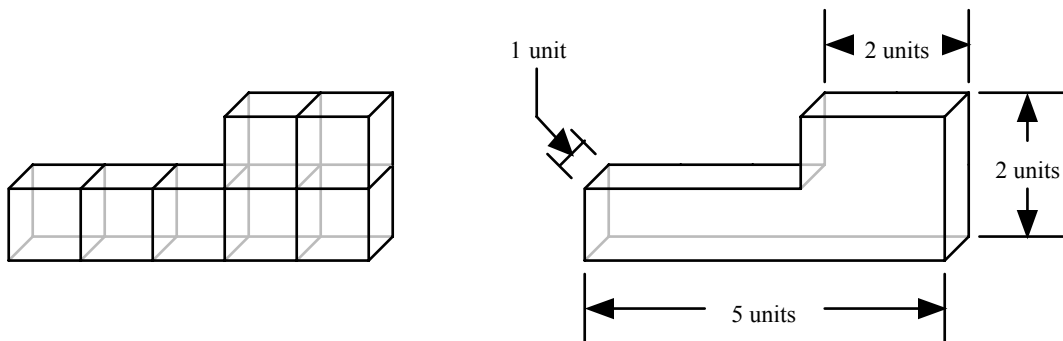


§6-3**VOLUME****Definition**

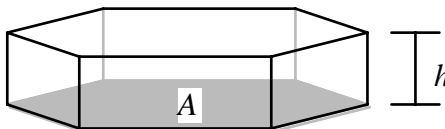
The **volume** of a three-dimensional solid is the number of cubic units contained within the solid.



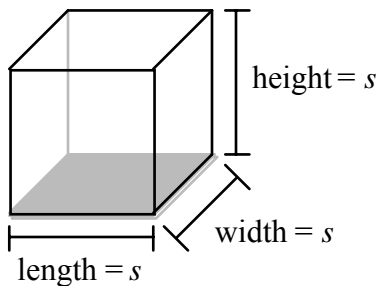
The volume of the solid shown above is 7 cubic units.

Formula

The **volume of a prism** with base area A and height h is given by: $V = Ah$

**Example 1**

What is the side length of a cube whose volume is 27 cubic inches.

**Solution**

All of the sides of a cube have the same length, s , so the area of the base is $A = s^2$ and the height is s . From the formula above, the volume is: $V = Ah = (s^2)(s) = s^3$.

$$\begin{aligned} V &= 27 \\ s^3 &= 27 \\ \sqrt[3]{s^3} &= \sqrt[3]{27} \\ s &= 3 \end{aligned}$$

So each side is 3 inches in length.

Example 2 A prism has a height of 12 cm and an irregular hexagonal base which is 23 square inches in area. Find the volume of the prism.

Solution $V = Ah = (23)(12) = 276$ cubic inches.

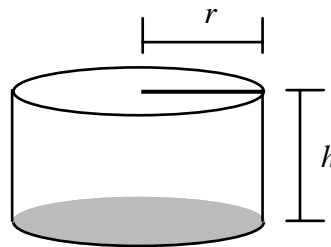
Example 3 Find the volume of a cylinder with a radius of 2 inches and height of 7 inches.

Solution Since the base of the cylinder is a circle, its area is $A = \pi r^2 = \pi(2)^2 = 4\pi$. Thus we have: $V = Ah = (4\pi)(7) = 28\pi$ cubic inches.

The process above leads to the following formula.

Formula

The **volume of a right circular cylinder** is given by: $V = \pi r^2 h$.



Example 4 A 10 foot high right circular cylindrical tank is completely filled with water. If the volume of the water is 27π cubic feet then what is the radius of the tank?

Solution

$$\begin{aligned} V &= \pi r^2 h \\ \frac{V}{\pi h} &= \frac{\pi r^2 h}{\pi h} \\ \frac{V}{\pi h} &= r^2 \\ \sqrt{\frac{V}{\pi h}} &= \sqrt{r^2} \\ \text{thus } r &= \sqrt{\frac{V}{\pi h}} \end{aligned}$$

So in this case $r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{27\pi}{10\pi}} = \sqrt{\frac{27}{10}} \approx 0.927$ feet.

Formula

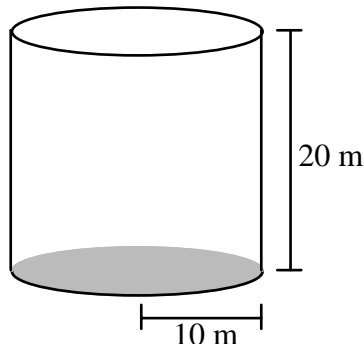
The **volume of a sphere** is given by: $V = \frac{4}{3} \pi r^3$, where r is the radius of the sphere.

Example 5 Calculate the volume of a sphere with a radius of 4 inches.

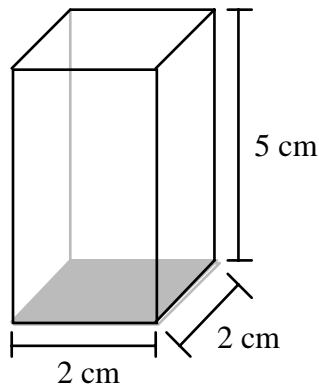
Solution $V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (4)^3 = \frac{256\pi}{3}$

Find the volume of each figure below.

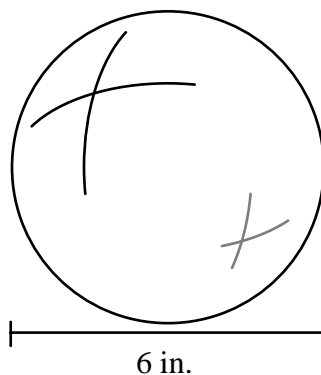
1. Right Circular Cylinder



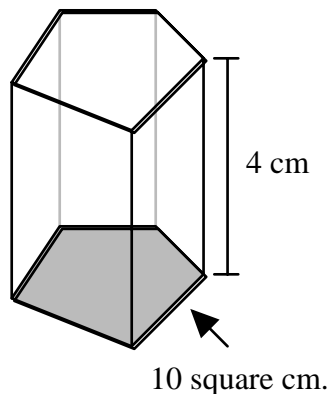
2. Right Square Prism



3. Sphere



4. Right Prism



Solve each word problem below.

5. A cylindrical building has a diameter of 350 feet and a height of 980 feet. What is the volume of this building?
6. A classroom is 31 feet long, 28 feet wide and 12 feet high. Find its volume.
7. The radius of the Earth is approximately 6.378×10^6 meters. What is the volume of the Earth?
8. A right triangular prism is 4 inches high. If the volume is 122 cubic inches, then what is the area of the base?
9. A nickel has a diameter of approximately 20 mm, and a \$2 stack of nickels is about 76 mm high. Find the volume of a single nickel.
10. A sphere and a cube have the same volume. If the sphere has a radius of 5 in., then what is the length of a side of the cube?

1. 2000π cu. m 2. 20 cu. cm 3. 36π cu. in. 4. 40 cu. cm
5. $30,012,500\pi$ cu. ft. 6. 10,416 cu. ft. 7. 1.09×10^{21} cu. m
8. 30.5 sq. in. 9. 190π cu. mm 10. $\sqrt[3]{\frac{500\pi}{3}}$ cu. in.