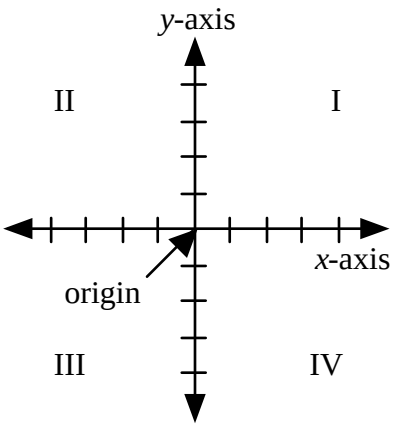
§5-1 COORDINATE GEOMETRY

Definition

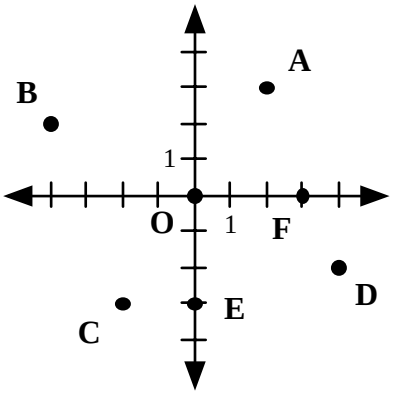
A **coordinate plane** consists of a plane divided into four **quadrants** (I, II, III and IV) by two perpendicular number lines called the ***x*-axis** and the ***y*-axis**. The point where the axes intersect is called the **origin**.



Example 1 Plot each of the following points on a coordinate plane:

A(2, 3), **B**(-4, 2), **C**(-2, -3), **D**(4, -2), **E**(0, -3), **F**(3, 0), **O**(0, 0)

Solution Plotting the points produces the plot below.



The results above illustrate the fact that the general location of a point can be determined by the signs of its coordinates. These results are summarized below.

Summary

***x*-coordinate**	***y*-coordinate**	**Location**
+	+	Quadrant I
-	+	Quadrant II
-	-	Quadrant III
+	-	Quadrant IV
Any	0	*x*-axis
0	Any	*y*-axis
0	0	Origin

Formula

The Midpoint Formula

The coordinates of the midpoint between two points in a plane (x_1, y_1) and (x_2, y_2) are given by:

$$\mathbf{M}\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

Formula

The Distance Formula

The distance between two points in a plane (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 2 Consider two points **A**(5, -3) and **B**(-1, 1).

a. Find the midpoint for these two points.

b. Find the distance between these two points.

Solution **a**. Let point A be the first point and point B be the second point.

$$\begin{aligned}\mathbf{M}\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right) &= \mathbf{M}\left(\frac{-1+5}{2}, \frac{1+(-3)}{2}\right)\\ &= \mathbf{M}\left(\frac{4}{2}, \frac{-2}{2}\right)\\ &= \mathbf{M}(2, -1)\end{aligned}$$

b. Let point A be the first point and point B be the second point.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\\ &= \sqrt{[(-1) - 5]^2 + [1 - (-3)]^2}\\ &= \sqrt{(-6)^2 + (4)^2}\\ &= \sqrt{36 + 16}\\ &= \sqrt{52}\\ &= \sqrt{4 \cdot 13}\\ &= 2\sqrt{13}\end{aligned}$$

In three dimensions, the midpoint formula and distance formula are slightly different.

Formula

The Midpoint Formula in Three Dimensions

The coordinates of the midpoint between two points in space (x_1, y_1, z_1) and (x_2, y_2, z_2) are given by:

$$\mathbf{M}\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}, \frac{z_2 + z_1}{2}\right)$$

Formula

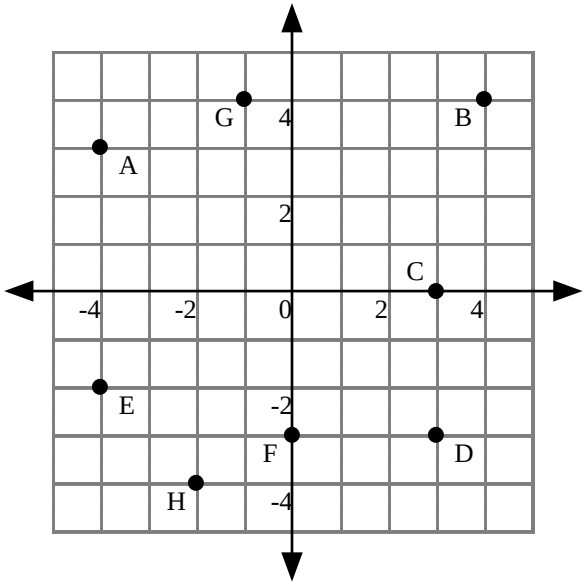
The Distance Formula in Three Dimensions

The distance between two points in space (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

§5-1 PROBLEM SET

Use the diagram below to answer the questions.



Write the coordinates for each point.

1. **A** **2**. **B** **3**. **C** **4**. **D**

5. **E** **6**. **F** **7**. **G** **8**. **H**

In which quadrant or on which axis is each point?

9. **A** **10**. **B** **11**. **C** **12**. **D**

13. **E** **14**. **F** **15**. (500, -20) **16**. (-35, -150)

Find the midpoint of the segment joining each pair of points.

17. **E** and **H** **18**. **C** and **G** **19**. **A** and **C** **20**. **F** and **D**

21. **C** and **D** **22**. **A** and **D** **23**. **A** and **E** **24**. **F** and **H**

25. (-69, 28) and (15, -7) **26**. (3, -4) and (15, 10)

27. (6, -3, 9) and (8, 7, 13) **28**. (60, -4, -30) and (51, 1, -37)

Find the distance between each pair of points. Simplify if possible.

29. **E** and **H** **30**. **C** and **G** **31**. **A** and **C** **32**. **F** and **D**

33. **C** and **D** **34**. **A** and **D** **35**. **A** and **E** **36**. **F** and **H**

37. (-69, 28) and (15, -7) **38**. (3, -4) and (15, 10)

39. (6, -3, 9) and (8, 7, 13) **40**. (60, -4, -30) and (51, 1, -37)

§5-1 PROBLEM SOLUTIONS

1.	(-4, 3)	**2**.	(4, 4)	**3**.	(3, 0)	**4**.	(3, -3)
5.	(-4, -2)	**6**.	(0, -3)	**7**.	(-1, 4)	**8**.	(-2, -4)
9.	Quadrant II	**10**.	Quadrant I	**11**.	*x*-axis	**12**.	Quadrant IV
13.	Quadrant III	**14**.	*y*-axis	**15**.	Quadrant IV	**16**.	Quadrant III
17.	(-3, -3)	**18**.	(1, 2)	**19**.	$\left(-\frac{1}{2}, \frac{3}{2}\right)$	**20**.	$\left(\frac{3}{2}, -3\right)$
21.	$\left(3, -\frac{3}{2}\right)$	**22**.	$\left(-\frac{1}{2}, 0\right)$	**23**.	$\left(-4, \frac{1}{2}\right)$	**24**.	$\left(-1, -\frac{7}{2}\right)$
25.	$\left(-27, \frac{21}{2}\right)$	**26**.	(9, 3)	**27**.	(7, 2, 11)	**28**.	$\left(\frac{111}{2}, -\frac{3}{2}, -\frac{67}{2}\right)$
29.	$2\sqrt{2}$	**30**.	$4\sqrt{2}$	**31**.	$\sqrt{58}$	**32**.	3
33.	3	**34**.	$\sqrt{85}$	**35**.	5	**36**.	$\sqrt{5}$
37.	91	**38**.	$2\sqrt{85}$	**39**.	$2\sqrt{30}$	**40**.	$\sqrt{155}$