§5-2 LINEAR RELATIONS

Definition

The **slope** of a line measures the direction that a line travels. It can be calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are two points on the line.

Lines with positive slope increase from left to right, while lines with negative slope decrease from left to right. Horizontal lines have slope zero while the slope of vertical lines is undefined.

Definition

The ***y*-intercept** of a line is the y coordinate of the point where the line crosses the y-axis.

There are two primary forms in which linear relations can be expressed.

Definition

The **slope-intercept form** for the equation of a line is $y = mx + b$ where m is the slope of the line and b is the y-intercept.

Definition

The **standard form** for the equation of a line is $Ax + By = C$ where A, B and C are real numbers.

Example 1 Find the slope and y-intercept for the line $2x + 3y = 9$.

Solution Solving for y changes the line into slope-intercept form.

$$2x + 3y = 9$$
$$3y = -2x + 9$$
$$y = -\frac{2}{3}x + 3$$

Thus the slope is $-\frac{2}{3}$ and the y-intercept is 3.

Example 2 Find the equation (in slope-intercept form) of the line that passes through the points (-2, 3) and (6, 7).

Solution The slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 3}{6 - (-2)} = \frac{4}{8} = \frac{1}{2}$

So the equation becomes $y = \frac{1}{2}x + b$.

Substituting the point (6, 7) into the equation produces: $7 = \frac{1}{2} \cdot 6 + b$

$$7 = 3 + b$$
$$7 - 3 = 3 + b - 3$$
$$4 = b$$

Therefore the line has the equation: $y = \frac{1}{2}x + 4$.

Property Two lines are **parallel** if they have the same slope. That is $m_1 = m_2$ where m_1 and m_2 are the slopes of the two lines.

Example 3 Find the equation (in slope-intercept form) of the line that is parallel to the line $y = \frac{1}{3}x - 2$ and passes through the point (-3, 1).

Solution The slope of the original line is $\frac{1}{3}$ so the slope of the new line must also be $\frac{1}{3}$.

So the equation for the new line is $y = \frac{1}{3}x + b$.

Substituting the point (-3, 1) into the equation produces:

$$\begin{aligned} 1 &= \frac{1}{3}\cdot(-3) + b \\ 1 &= -1 + b \\ 1 + 1 &= -1 + b + 1 \\ 2 &= b \end{aligned}$$

Therefore the line has the equation: $y = \frac{1}{3}x + 2$.

Property Two lines are **perpendicular** if their slopes are negative reciprocals of each other. That is, $m_1 = -\frac{1}{m_2}$ where m_1 and m_2 are the slopes of the two lines.

Example 4 Find the equation (in slope-intercept form) of the line that is perpendicular to the line $y = \frac{1}{3}x - 2$ and passes through the point (-3, 1).

Solution The slope of the original line is $\frac{1}{3}$ so the slope of the new line must be -3.

So the equation for the new line is $y = -3x + b$.

Substituting the point (-3, 1) into the equation produces:

$$\begin{aligned} 1 &= -3\cdot(-3) + b \\ 1 &= 9 + b \\ 1 - 9 &= 9 + b - 9 \\ -8 &= b \end{aligned}$$

Therefore the line has the equation: $y = -3x - 8$.

§5-2 PROBLEM SET

Find the slope and y-intercept for each line below

1. $4x - 2y = 5$

2. $y - 2x = 4$

3. $y = \frac{1}{4}x + 7$

4. $\frac{1}{2}y = \frac{1}{3}x + 2$

5. $y + 3x = -1$

6. $3x + 2y = 4$

7. $5x - 3y = 0$

8. $1 = \frac{2}{3}y + \frac{1}{5}x$

9. $2x + 5y = 10$

10. $2x + 3y = 6$

11. $y = 5$

12. $3x - \frac{5}{6}y + 2 = 0$

Write an equation in slope-intercept form for each of the following lines.

13. The line with a slope of $\frac{1}{3}$ and a y-intercept of 13.

14. The line with a slope of -3 and a y-intercept of 4.

15. The line with a slope of 5 which passes through the point (1, 3).

16. The line with a slope of -2 which passes through the point (4, -1).

17. The line which passes through the points (-2, -5) and (0, 1).

18. The line which passes through the points (5, -2) and (-3, 4).

19. The line which is parallel to $y = -2x + 7$ and passes through the point (2, 10).

20. The line which is parallel to $y = -\frac{2}{3}x + 1$ and passes through the point (2, 3).

21. The line which is perpendicular to $2x - y = -3$ and passes through the point (3, 0).

22. The line which is perpendicular to $y = \frac{1}{2}x - 3$ and passes through the point (1, 5).

23. The line which is parallel to $y = 5$ and passes through the point (4, -2).

24. The line which is perpendicular to $y = -2$ and passes through the point (-1, -3).

25. The line which is perpendicular to $x = 4$ and passes through the point (-4, 1).

26. The line which is parallel to $x = -1$ and passes through the point (-5, -3).

27. A line segment has endpoints (-5, 4) and (13, -2). Find the equation of the line which is perpendicular to this segment and which passes through the midpoint of the line segment.

§5-2 PROBLEM SOLUTIONS

1. $m = 2;\ b = -\frac{5}{2}$

2. $m = 2; b = 4$

3. $m = \frac{1}{4}; b = 7$

4. $m = \frac{2}{3}; b = 4$

5. $m = -3; b = -1$

6. $m = -\frac{3}{2}; b = 2$

7. $m = \frac{5}{3}; b = 0$

8. $m = -\frac{3}{10}; b = \frac{3}{2}$

9. $m = -\frac{2}{5}; b = 2$

10. $m = -\frac{2}{3}; b = 2$

11. $m = 0; b = 5$

12. $m = \frac{18}{5};\ b = \frac{12}{5}$

13. $y = \frac{1}{3}x + 13$

14. $y = -3x + 4$

15. $y = 5x - 2$

16. $y = -2x + 7$

17. $y = 3x + 1$

18. $y = -\frac{3}{4}x + \frac{7}{4}$

19. $y = -2x + 14$

20. $y = -\frac{2}{3}x + \frac{13}{3}$

21. $y = -\frac{1}{2}x + \frac{3}{2}$

22. $y = -2x + 7$

23. $y = -2$

24. $x = -1$

25. $y = 1$

26. $x = -5$

27. $y = 3x - 11$