§5-4 FUNCTIONS

Definition

> A **function** is a rule that assigns a unique real number to each number in a specified set of real numbers.

Functions are expressed in the form $f(x) = u$ where u is a variable expression. $f(x)$ does not indicate that f is multiplied times x but rather that the function f should be evaluated at the value x.

Example 1 Evaluate the function $f(x) = 2x^2 + 11x - 7$ at the indicated values.

a. $x = 2$ **b**. $x = -1$ **c**. $x = t$ **d**. $x = b + 1$

Solution

a. $f(2) = 2(2)^2 + 11(2) - 7 = 2(4) + 22 - 7 = 8 + 15 = 23$

b. $f(-1) = 2(-1)^2 + 11(-1) - 7 = 2(1) - 11 - 7 = 2 - 18 = -16$

c. $f(t) = 2(t)^2 + 11(t) - 7 = 2t^2 + 11t - 7$

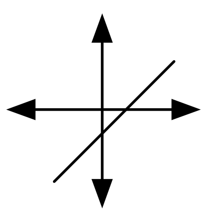
d. $f(b + 1) = 2(b+1)^2 + 11(b+1) - 7 = 2(b^2 + 2b + 1) + 11b + 11 - 7$
$= 2b^2 + 4b + 2 + 11b + 4 = 2b^2 + 15b + 6$

Theorem

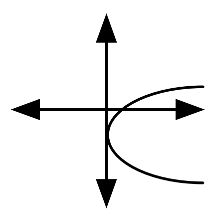
> **<u>The Vertical Line Test</u>**
> If a vertical line drawn anywhere on the graph of a relation (in x and y) will intersect the graph at no more than one point, then the relation is a function of x.

Example 2 Use the vertical line test to determine which of the following relations are functions of x.

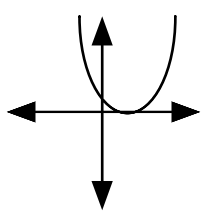
a.



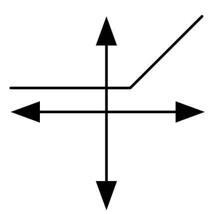
b.



c.



d.



Solution

a. This is a function since any vertical line will intersect the graph at one point.

b. This is not a function many vertical lines will intersect the graph at two points.

c. This is a function since any vertical line will intersect the graph at one point.

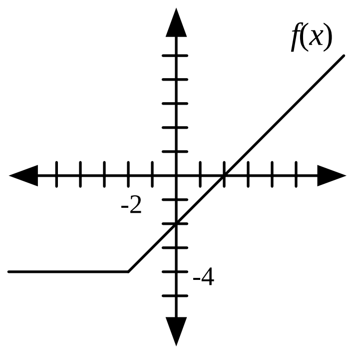
d. This is a function since any vertical line will intersect the graph at one point.

Functions can be *translated* (moved) by using the following rules.

Properties

Function	Description
$g(x) = f(x) + k$	$g(x)$ is $f(x)$ translated k units up
$g(x) = f(x) - \text{k}$	$g(x)$ is $f(x)$ translated k units down
$g(x) = f(x - h)$	$g(x)$ is $f(x)$ translated h units to the right
$g(x) = f(x + h)$	$g(x)$ is $f(x)$ translated h units to the left

Example 3 Use the graph of $f(x)$ below to graph each of the following functions.



a. $g(x) = f(x) + 3$

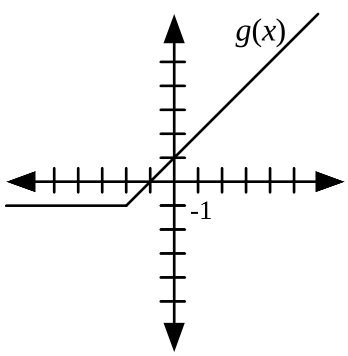
b. $h(x) = f(x) - 1$

c. $u(x) = f(x - 3)$

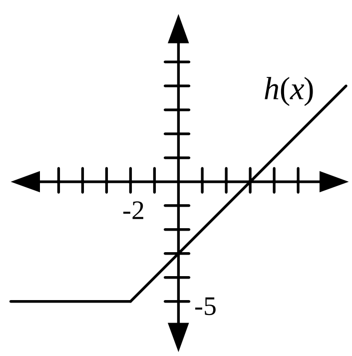
d. $v(x) = f(x + 2)$

Solution

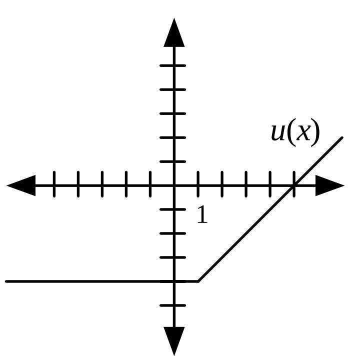
a. $g(x)$ is $f(x)$ translated 3 units up.



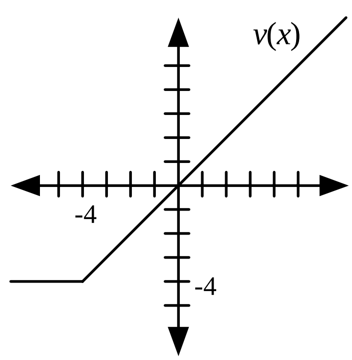
b. $h(x)$ is $f(x)$ translated 1 unit down.



c. $u(x)$ is $f(x)$ translated right 3 units.



d. $v(x)$ is $f(x)$ translated left 2 units.



§5-4 PROBLEM SET

Evaluate each function at the indicated values.

1. $f(x) = 2x + 5$; $x = -3$

2. $g(x) = x^2 - 1$; $x = 2$

3. $h(t) = |t - 2| + 1$; $t = -3$

4. $f(a) = \dfrac{a+1}{3a-7}$; $a = 3$

5. $g(x) = 5 - \sqrt{x+2}$; $x = 14$

6. $h(x) = 2^x + 1$; $x = 4$

7. $u(r, s) = \dfrac{2r+1}{s}$; $r = 5$, $s = -2$

8. $g(x, y) = xy - 2y + \sqrt{y}$; $x = -1$, $y = 4$

Evaluate the function $f(x)$ at the indicated values: $f(x) = \dfrac{2x+2}{2x-3}$

9. $f(0)$ **10.** $f(3)$ **11.** $f(a)$ **12.** $f(x + 1)$

Evaluate the function $g(x)$ at the indicated values: $g(x) = 2x^2 + x - 2$

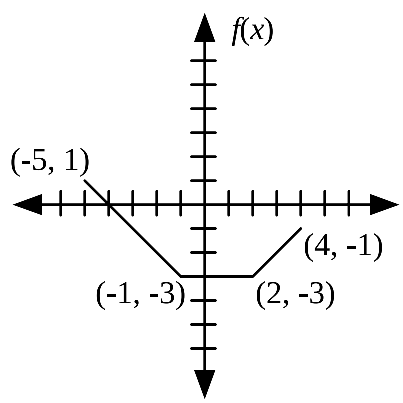
13. $g(0)$ **14.** $g(3)$ **15.** $g(a)$ **16.** $g(x + 1)$

Use the function $f(x)$ to determine $g(x)$ in each case: $f(x) = 3x^2 - x - 1$.

17. $g(x) = f(x) + 5$ **18.** $g(x) = f(x) - 1$ **19.** $g(x) = f(x - 3)$

20. $g(x) = f(x + 2)$ **21.** $g(x) = f(x - 1) + 2$ **22.** $g(x) = f(x + 2) - 3$

Use the graph of the function $f(x)$ to draw the graph of $g(x)$ in each case.



23. $g(x) = f(x) + 3$ **24.** $g(x) = f(x) - 2$ **25.** $g(x) = f(x + 1)$

26. $g(x) = f(x - 2)$ **27.** $g(x) = f(x - 1) + 2$ **28.** $g(x) = f(x + 2) - 1$

1. -1 **2.** 3 **3.** 6 **4.** 2 **5.** 1

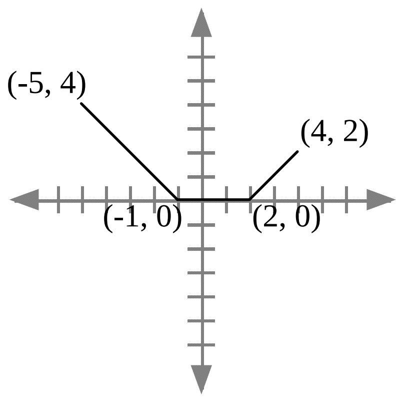
6. 17 **7.** $-\frac{11}{2}$ **8.** -10 **9.** $-\frac{2}{3}$ **10.** $\frac{8}{3}$

11. $\frac{2a+2}{2a-3}$ **12.** $\frac{2x+4}{2x-1}$ **13.** -2 **14.** 19 **15.** $2a^2+a-2$

16. $2x^2+5x+1$ **17.** $3x^2-x+4$ **18.** $3x^2-x-2$ **19.** $3x^2-19x+29$

20. $3x^2+11x+9$ **21.** $3x^2-7x+5$ **22.** $3x^2+11x+6$

23.



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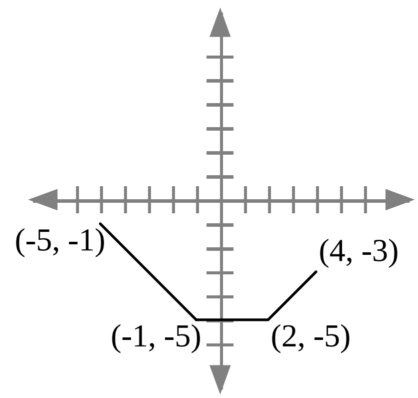

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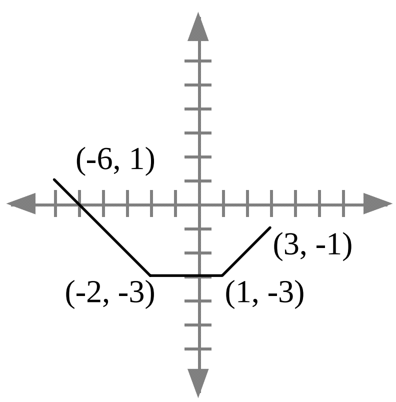

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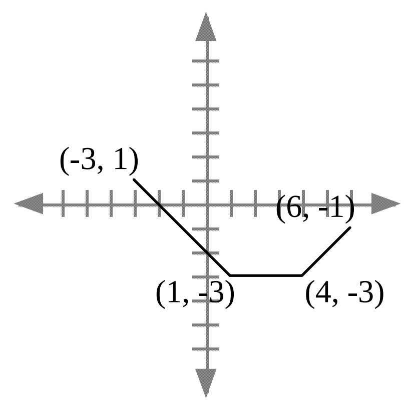

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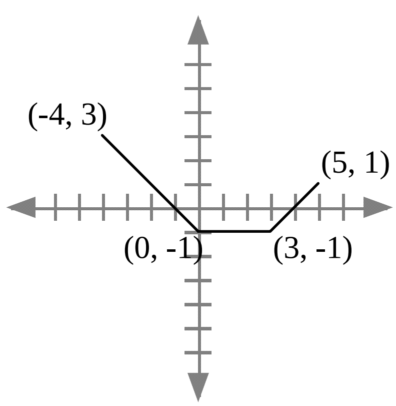

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