

**§2-1****INTEGER EXPONENTS****Definition**

$a$  raised to the  $n$ th **power**, written as  $a^n$ , means  $a$  multiplied times itself  $n$  times,  $a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors of } a}$ , where  $a$  is called the **base** and  $n$  is called the **exponent**.

The expression  $3^4$  is read “3 to the fourth power” and means  $3 \times 3 \times 3 \times 3 = 81$ .

**Properties**

<u>Properties of Exponents</u>	<u>Examples</u>
$a^m a^n = a^{m+n}$	$x^3 x^2 = x^5$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^7}{x^4} = x^3$
$(a^m)^n = a^{mn}$	$(x^3)^2 = x^6$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{2x}{y}\right)^2 = \frac{4x^2}{y^2}$
$a^{-m} = \frac{1}{a^m}$	$x^{-3} = \frac{1}{x^3}$
$\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$	$\left(\frac{2x}{y}\right)^{-2} = \frac{y^2}{4x^2}$
$a^0 = 1$ , where $a \neq 0$	$(-2x^3)^0 = 1$ , where $x \neq 0$

**Example 1** Simplify the expression  $(2x^2y^{-3})^{-2}$ .

**Solution**

$$\begin{aligned} (2x^2y^{-3})^{-2} &= \frac{1}{(2x^2y^{-3})^2} \\ &= \frac{1}{2^2(x^2)^2(y^{-3})^2} \\ &= \frac{1}{4x^4y^{-6}} \\ &= \frac{y^6}{4x^4} \end{aligned}$$

**Example 2** Simplify the expression  $\left(\frac{x^3y^{-2}}{x^{-2}y^{-1}}\right)^{-1}$ .

**Solution**

$$\begin{aligned} \left(\frac{x^3y^{-2}}{x^{-2}y^{-1}}\right)^{-1} &= \frac{x^{-3}y^2}{x^2y^1} \\ &= \frac{y^2}{x^3x^2y} \\ &= \frac{y}{x^5} \end{aligned}$$

**Example 3** Simplify the expression  $x^{-2} \div xy^{-3}$ .

**Solution**

$$\begin{aligned}x^{-2} \div xy^{-3} &= \frac{1}{x^2} \div \frac{x}{y^3} \\ &= \frac{1}{x^2} \cdot \frac{y^3}{x} \\ &= \frac{y^3}{x^3}\end{aligned}$$

**Example 4** Simplify the expression  $\frac{2x^{-3}y^2z^{-1}}{(xy^3z^{-2})^{-1}}$ .

**Solution**

$$\begin{aligned}\frac{2x^{-3}y^2z^{-1}}{(xy^3z^{-2})^{-1}} &= \frac{2x^{-3}y^2z^{-1}}{x^{-1}y^{-3}z^2} \\ &= \frac{2xy^2y^3}{x^3z^2z} \\ &= \frac{2xy^5}{x^3z^3}\end{aligned}$$

**Definition**

A number is in **scientific notation** when it is written in the form  $a \times 10^n$  where  $1 \leq a < 10$  and  $n$  is an integer.

**Example 5** Convert 12,000 to scientific notation.

**Solution**

$$\begin{aligned}12,000 &= 1.2 \times 10,000 \\ &= 1.2 \times 10^4\end{aligned}$$

**Example 6** Convert -0.0015 to scientific notation.

**Solution**

$$\begin{aligned}-0.0015 &= -1.5 \div 1000 \\ &= -1.5 \div 10^3 \\ &= -1.5 \times 10^{-3}\end{aligned}$$

**Example 7** Convert  $6.35 \times 10^4$  to decimal notation.

**Solution**

$$\begin{aligned}6.35 \times 10^4 &= 6.35 \times 10,000 \\ &= 63,500\end{aligned}$$

**Example 8** Convert  $8.41 \times 10^{-5}$  to decimal notation.

**Solution**

$$\begin{aligned}8.41 \times 10^{-5} &= 8.41 \div 10^5 \\ &= 8.41 \div 100,000 \\ &= 0.0000841\end{aligned}$$

**Example 9** Multiply  $(2.5 \times 10^{-5})(4.0 \times 10^8)$  and express your answer in scientific notation.

**Solution**

$$\begin{aligned}(2.5 \times 10^{-5})(4.0 \times 10^8) &= (2.5 \cdot 4.0)(10^{-5} \cdot 10^8) \\ &= 10 \times 10^3 \\ &= 1.0 \times 10^4\end{aligned}$$

Simplify each expression.

- |     |  |     |  |     |  |     |  |
|-----|--|-----|--|-----|--|-----|--|
| 1.  | $(a^2b)(ab^4)$                               | 2.  | $(-3x^2y)^3$                                       | 3.  | $[(2x)^3]^2$                                 | 4.  | $(2rs^4)(r^2s)^4$                            |
| 5.  | $x^{2n} \cdot x^n$                           | 6.  | $(r^3s^{-1})^{-2}$                                 | 7.  | $a^{-5} \cdot a^3$                           | 8.  | $\frac{x^{-3}y^4}{x^4y^9}$                   |
| 9.  | $\frac{(2a^{-1}b^2)^2}{(a^{-2}b)^3}$         | 10. | $\left(\frac{x^2y^{-3}}{x^{-1}y^{-2}}\right)^{-2}$ | 11. | $(-2xy^{-2})^3$                              | 12. | $(-2xy^{-2})^2$                              |
| 13. | $\frac{(3x^{-1}y^2z^{-2})^{-2}}{2x^2z^{-2}}$ | 14. | $[(2a^{-1}b^2)^{-2}]^{-1}$                         | 15. | $\frac{4a^{-2}b^3c^{-1}}{(ab^2c^{-2})^{-2}}$ | 16. | $\frac{(2a^2c^{-2})^{-3}}{(a^2c^{-1})^{-2}}$ |
| 17. | $\frac{a^{-3}}{a^{-4}}$                      | 18. | $\frac{(c^{-2})^{-3}}{(c^{-1})^{-2}}$              | 19. | $\frac{(x-y)^{-3}}{(x-y)^2}$                 | 20. | $4x^{-2} + y - (2^{-1}x)^{-2}$               |
| 21. | $\frac{(a+b)^{-3}}{(a+b)^{-2}}$              | 22. | $\left(a + \frac{1}{b} - b^{-1}\right)^2$          | 23. | $\frac{(2^x)^y}{(2^y)^x}$                    | 24. | $\frac{(2x)^0}{(a+b)^{-1}}, x \neq 0$        |
| 25. | $\frac{[(-2)^{-1}]^3}{(2^3)^{-1}}$           | 26. | $x(x^{-1} + x)$                                    | 27. | $(x^{-1} - y^{-1})^{-1}$                     | 28. | $\frac{(a+b^0)^{-6}}{(a+1)^{-5}}, b \neq 0$  |

Express each quantity in scientific notation.

- |     |          |     |               |     |           |
|-----|----------|-----|---------------|-----|-----------|
| 29. | 3500     | 30. | 0.004         | 31. | 4,612,000 |
| 32. | 0.000056 | 33. | 4,000,000,000 | 34. | 40.04     |
| 35. | -123.123 | 36. | -2382         | 37. | -0.00234  |

Express each quantity in decimal notation.

- |     |                       |     |                     |     |                         |
|-----|-----------------------|-----|---------------------|-----|-------------------------|
| 38. | $2 \times 10^3$       | 39. | $3 \times 10^{-4}$  | 40. | $1.53 \times 10^5$      |
| 41. | $9.75 \times 10^{-3}$ | 42. | $4.23 \times 10^8$  | 43. | $-0.001 \times 10^{-3}$ |
| 44. | $-8.75 \times 10^0$   | 45. | $-3.26 \times 10^3$ | 46. | $-3.26 \times 10^{-3}$  |

Perform the indicated operation and express your answer in scientific notation.

- |     |                                  |     |  |     |  |
|-----|----------------------------------|-----|--|-----|--|
| 47. | $(2 \times 10^5)(3 \times 10^7)$ | 48. | $\frac{12 \times 10^{-4}}{4 \times 10^2}$          | 49. | $(3.2 \times 10^{-4})(5 \times 10^{-6})$ |
| 50. | $\frac{0.0000049}{0.00007}$      | 51. | $\frac{0.012 \times 10^{-7}}{3.0 \times 10^{-13}}$ | 52. | $(5,000,000)(0.00002)$                   |

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|----------------------------|--------------------------|----------------------------|------------------------|
| 1. $a^3b^5$                | 2. $-27x^6y^3$           | 3. $64x^6$                 | 4. $2r^9s^8$           |
| 5. $x^{3n}$                | 6. $\frac{s^2}{r^6}$     | 7. $\frac{1}{a^2}$         | 8. $\frac{1}{x^7y^5}$  |
| 9. $4a^4b$                 | 10. $\frac{y^2}{x^6}$    | 11. $-\frac{8x^3}{y^6}$    | 12. $\frac{4x^2}{y^4}$ |
| 13. $\frac{z^6}{18y^4}$    | 14. $\frac{4b^4}{a^2}$   | 15. $\frac{4b^7}{c^5}$     | 16. $\frac{c^4}{8a^2}$ |
| 17. $a$                    | 18. $c^4$                | 19. $\frac{1}{(x-y)^5}$    | 20. $y$                |
| 21. $\frac{1}{a+b}$        | 22. $a^2$                | 23. $1$                    | 24. $a+b$              |
| 25. $-1$                   | 26. $1+x^2$              | 27. $\frac{xy}{y-x}$       | 28. $\frac{1}{a+1}$    |
| 29. $3.5 \times 10^3$      | 30. $4 \times 10^{-3}$   | 31. $4.612 \times 10^6$    |                        |
| 32. $5.6 \times 10^{-5}$   | 33. $4 \times 10^9$      | 34. $4.004 \times 10^1$    |                        |
| 35. $-1.23123 \times 10^2$ | 36. $-2.382 \times 10^3$ | 37. $-2.34 \times 10^{-3}$ |                        |
| 38. $2000$                 | 39. $0.0003$             | 40. $153,000$              |                        |
| 41. $0.00975$              | 42. $423,000,000$        | 43. $-0.000001$            |                        |
| 44. $-8.75$                | 45. $-3260$              | 46. $-0.00326$             |                        |
| 47. $6 \times 10^{12}$     | 48. $3 \times 10^{-6}$   | 49. $1.6 \times 10^{-9}$   |                        |
| 50. $7 \times 10^{-2}$     | 51. $4 \times 10^3$      | 52. $1 \times 10^2$        |                        |